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RESEARCH ARTICLE

Bubbly Bitcoin

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Abstract
There has been a burgeoning Fintech literature in the past years, especially on cryptocurrencies. However, there is lack of research handling cryptocurrencies in a mainstream macroeconomic model. To bridge the gap, we develop a model for Bitcoin-like cryptocurrency as risky and costly bubbles in an infinite-horizon production economy. This model is consistent with the following facts: (1) the surging Bitcoin market presents enormous volatility, (2) its price dynamics are significantly sensitive to both market sentiment and policy stances. Entrepreneurial firms choose to hold Bitcoins as liquid assets to buffer idiosyncratic investment distortions. The intrinsically worthless Bitcoins can emerge as rational bubbles when the market sentiment is optimistic enough. On the one hand, bubbly Bitcoins provide market liquidity to facilitate investment in the real sector, while on the other hand, they deteriorate the investment efficiency and crowd out aggregate production. Our quantitative exercise produces various cyclical features of Bitcoin bubbles and find that the collapse of Bitcoin bubbles can improve social welfare by decreasing distortion-driven real investment.

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"The market that looks most like a bubble to me is Bitcoin and its brethren."
—Richard Thaler

1 Introduction

Bitcoin is a digital currency backed by no government but a scarcity that is mathematically predetermined. A fixed number of 21 million Bitcoins can be released into circulation through a process called "mining", whereby highly-powered computers around the world compete to solve cryptographic tasks that are key to recording Bitcoin transactions. The price of Bitcoin is the dollar value of 1 unit of Bitcoin, in the same way that 1 pound exchanges for a certain amount of dollars.

Since the inception of Bitcoin in 2009, the price of this digital money has gone from less than a dollar to hit approximately $20,000 at the end of 2017. It then suffered a 38% slump within one week in November 2018, crashing later to under $4,000. More upswings and bursts followed, only to see the Bitcoin price surge to $40,000 on January 9th 2021, but then again slumping 20% in the next 5 days (see Fig. 1). The enormous volatility in the Bitcoin price is hard to square with standard (e.g. random-matching) models of money, and naturally increases the number of critics who claim that cryptocurrencies are speculative bubbles, for which a burst may lead to systemic risk. To make the matter more pertinent, since the production of Bitcoin-like cryptocurrencies is costly, there has been increasing concern regarding whether the cryptocurrencies are a pure waste of resources (Williamson 2018).

Motivated by these concerns, it is natural for both academics and policy makers to explore the possible causes and macroeconomic consequences of Bitcoin-like cryptocurrencies as costly and risky bubbles. However, there is a surprisingly limited number of macroeconomic analyses on this topic. To this end, we construct a macroeconomic model with cryptocurrencies endogenously emerging as rational bubbles, but are risky and costly to produce.

We begin by highlighting key empirical regularities regarding the Bitcoin, market sentiment, policy regulations, and the real economy. Anecdotal evidence implies that large price changes in the Bitcoin market are usually associated with big events related to policy tightening or market turbulence. Our further empirical analysis reveals that negative sentiment predicts busts in future Bitcoin prices. Moreover, we show that policy tightening significantly deteriorates sentiment and raises the probability of collapse in future Bitcoin prices.

To unlock those empirical patterns, we construct a dynamic general equilibrium model of the Bitcoin market structure. We then use the model to study the macroeconomic consequences of the Bitcoin market turbulence. Our quantitative exercise could be treated as a counterfactual analysis which is ahead of the time. We aim to convey that if in the future the scale of bitcoin or cryptocurrency market become comparable to traditional asset markets, what would happen to the aggregate economy when its

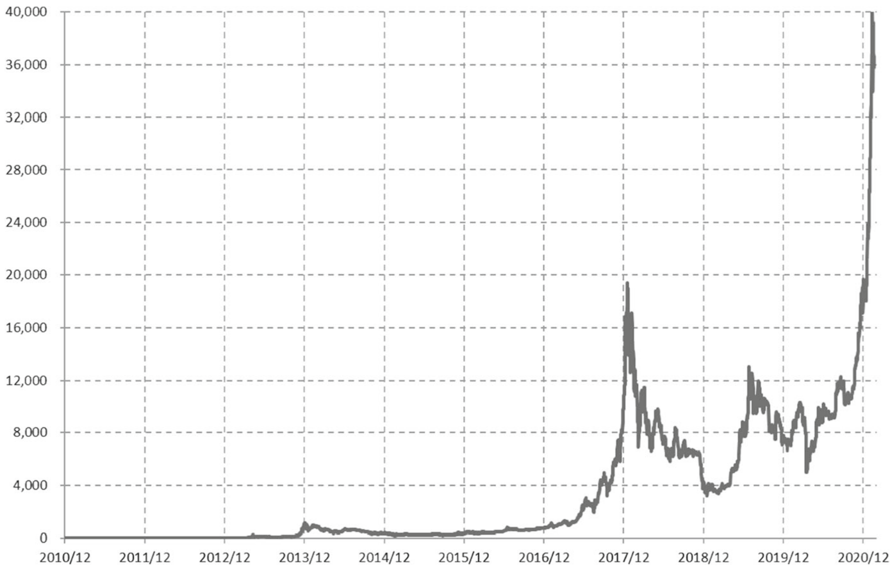


Fig. 1 Daily Bitcoin Price in US Dollar . Data Source: CoinMarketCap.com

price collapses. We believe this question not only interests the academics but also the financial regulators and policy makers.

The model features heterogeneous entrepreneurial firms (or investors), who face idiosyncratic investment distortion shock that directly affects the firms' investment cost. Due to the liquidity constraints (Miao et al. 2015a), investment distortion shocks are uninsurable and thus incur idiosyncratic uncertainty, which increases with the dispersion of shocks. This intrinsically generates demand for Bitcoin as a store of value. When real investment is costly, firms opt to hoard liquidity in the form of Bitcoin for possible favorable shocks in the future. Therefore, the Bitcoin bears a positive liquidity premium. Moreover, Bitcoins are resource consuming since they are produced by miners through mining activities via a proof-of-work mechanism.[1] To model the market risks, we assume that the value of the Bitcoin bubble would vanish with a positive probability. We interpret the inverse of this probability as the market sentiment in the model. A rise in this probability of bubble burst indicates that the market sentiment has become more pessimistic or the policy has become tighter. We show that the intrinsically worthless Bitcoin may emerge endogenously as a rational bubble only when the dispersion of investment distortion shocks is sufficiently large and market sentiment is sufficiently optimistic.

In our model, the Bitcoin bubble facilitates market liquidity and thus boosts real investment, but in the meantime, it deteriorates investment efficiency and crowds out real resources. The overall effects of the Bitcoin bubble on the real economy depend on which force dominates. To further evaluate the consequences of a change in public sentiment on the Bitcoin market and the real economy, we conduct a quantitative

[1] See the finance literature discussed in our literature review for details on the market structure of blockchain and mining activities.

analysis based on the baseline model of the Bitcoin bubble. The steady-state analysis suggests that the relationship between market sentiment and the aggregate output (or investment) has an inverted-U shape. When sentiment is positive, i.e., the probability of a bubble burst is relatively low, a deterioration in market sentiment may stimulate the aggregate investment and output. However, if sentiment is more negative (or the probability of a bubble burst is relatively high), a deterioration in the market sentiment may depress the aggregate economy. In either case, a more pessimistic sentiment unambiguously dampens the steady-state Bitcoin price. The dynamic analysis suggests that the market sentiment has positive impact on Bitcoin price dynamics and the cyclicality relationship between the Bitcoin price and the aggregate economy is ambiguous depending on the level of market sentiment.

Both Japan in the early 1990s and the US in the recent Great Recession have witnessed the collapse of housing and stock bubbles. To this end, it is natural for us to ask what would happen if the risky Bitcoin bubble collapses. The transition dynamics of this counterfactual analysis demonstrate that the burst of the Bitcoin bubble will lead to liquidity dry-up and thus impede investment and output. As it turns out, the quantitative performance heavily relies on the magnitude of investment distortions, which determines the bubble size. In contrast, aggregate consumption increases because the collapse of the bubble alleviates the crowding-out effect caused by Bitcoin production. As a consequence, the burst of the bubble improves social welfare. When the size of the bubble is large, the welfare improvement turns out to be substantial.

Related literature Our paper is largely motivated by the burgeoning Fintech literature, especially those on cryptocurrencies.[2] Yermack (2015) is among the first academic papers to address cryptocurrency. Recent progress on empirical facts regarding cryptocurrency investments and price determination includes Bianchi (2017), Stoffels (2017), Borri (2018), Borri and Shakhnov (2018), Foley et al. (2018), Hu et al. (2018), Li et al. (2018) and Liu and Tsyvinski (2018) .

The finance literature on modeling cryptocurrencies has burgeoned. See Weber (2016), Huberman et al. (2017), Biais et al. (2018), Catalini and Gans (2016), Chiu and Koeppl (2017), Cong and He (2018), Cong et al. (2018a), Cong et al. (2018b), Davidson et al. (2016), Sockin and Xiong (2018), Saleh (2018), Schilling and Uhlig (2018), Abadi and Brunnermeier (2018), Routledge et al. (2018), and Makarov and Schoar (2018), among others for theoretical analyses of cryptocurrencies. Most of those papers focus on the market microstructure of Blockchain and mining activities.

However, there is surprisingly a limited number of macroeconomic analyses on cryptocurrencies. The exceptions include Chiu and Koeppl (2017) and Schilling and Uhlig (2018). Our paper emphasizes the role of cryptocurrency as the store of value while these two papers focus on the role of payment. Specifically, our framework follows Bewley (1983), in which the agents hold idle liquidity (cryptocurrency) as a buffer when they are facing uninsurable idiosyncratic uncertainty in an incomplete market. Whereas, Chiu and Koeppl (2017) and Schilling and Uhlig (2018) adopt the day-night frictional money market structure in Lagos and Wright (2005), where

[2] See Hilary and Liu (2018) for an early review of the Fintech literature related to cryptocurrencies.

agents hold money as a medium of exchange for the potential opportunity of purchasing consumption. In this regard, our theory of cryptocurrency is complementary to their models. The modeling strategy in our paper allows us to incorporate the cryptocurrency into a mainstream macroeconomic model with production economy. Under this framework, we could study the dynamic interactions between the macroeconomic fluctuations and the price volatility of cryptocurrency both theoretically and quantitatively. Besides, with a setup of heterogeneous investors in a production economy, we could document how the cryptocurrency affects the real economy through resource allocation efficiency, which is of general interest from a macroeconomic perspective.

To the best of our knowledge, our paper is the first to adopt the bubble approach to theoretically and quantitatively characterize cryptocurrencies. In addition to our own empirical findings, our model on bubbly cryptocurrencies is partly motivated by the empirical analysis by Liu and Tsyvinski (2018). They find that only momentum and the proxies for investor attention consistently explain the variations of cryptocurrency returns, which suggests that markets do not view cryptocurrencies as being similar to standard classes of assets. Instead, market sentiment, a key element of asset bubbles, plays a crucial role in the price dynamics of cryptocurrencies, as shown in our empirical, theoretical and quantitative analyses.

More broadly speaking, our paper belongs to the literature on asset bubbles. We now briefly discuss the relationship to the literature. The early works on asset bubbles include Samuelson (1958) and Tirole (1985), who developed overlapping-generation models. In addition, see Weil (1987) for an introduction of stochastic bubbles. The recent progress made on bubble theory consists of two branches of research. On the one hand, the works that consider the OLG framework include Martin and Ventura (2012), Farhi and Tirole (2011), Chen and Wen (2017) and Bengui and Phan (2018). On the other hand, for an infinite-horizon framework, see Kocherlakota (2009), Wang and Wen (2012b), Aoki and Nikolov (2015), Hirano and Yanagawa (2016) , Miao et al. (2015b), Dong et al. (2018) and Miao and Wang (2018) for firm bubbles. See Miao (2014) and Martin and Ventura (2018) for comprehensive surveys on rational bubbles.

The most related paper is Miao et al. (2015a). They show that under some conditions, land bubbles may crowd out consumption and thus lower social welfare. They also find that property taxes, Tobin's taxes, macro-prudential policy, and credit policy can prevent the formation of a land bubble. Our paper is built off a variant of theirs. The main differences are as below. First, Miao et al. (2015a) do not consider the stochastic bubble, and therefore, they do not characterize the systemic risk of asset bubbles. Secondly, the bubbly assets in our model are not controlled or backed by the government, unlike fiat money, government bonds, and housing titles or land titles. Moreover, the setup in our paper shows that the bubbly assets are sensitive to the collective belief of the assets and the collective belief of whether the government would make access to the assets more or less costly, making it a risky store of value. Thirdly, not only Miao et al. (2015a) but also almost the whole literature on asset bubbles treats the supply of bubbles as exogenously given. However, the Bitcoin-like bubble is produced by miners through the mining activities. Therefore, as motivated by the aforementioned finance literature, we model cryptocurrencies as costly bubbles, which calls for the input of real resources (mining). Finally, we are among the first to document the aggregate effects of bubbly cryptocurrencies at length in both theoretical and quantitative analyses.

The rest of the paper proceeds as follows. Section 2 discusses the empirical findings regarding the relationship between market sentiment (or policy stances) and the price dynamics of the Bitcoin market. Section 3 presents a dynamic general equilibrium model with the Bitcoin market structure. Section 4 characterizes individuals' optimal decisions and the properties of bubbly and bubbleless equilibria. Section 5 calibrates the baseline model and conducts the quantitative analysis. Section 6 concludes. Data descriptions, proofs and more empirical and quantitative results are provided in the Appendices.

2 Facts about Bitcoin price dynamics

In this section, we discuss anecdotal evidence on Bitcoin price dynamics. We then use a statistical analysis to document several stylized facts regarding the impacts of Bitcoin regulations and market sentiment on Bitcoin prices.

2.1 Anecdotal analysis

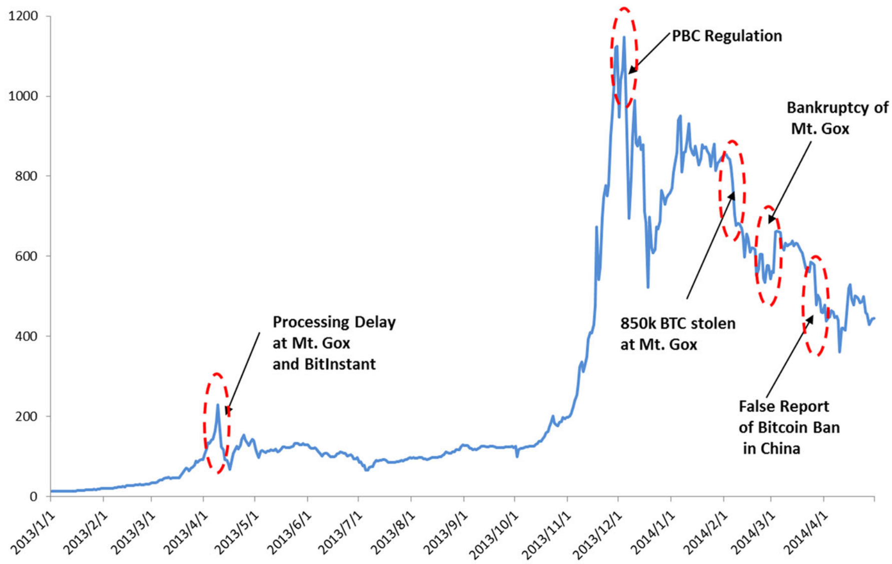
Bitcoin prices are extremely volatile. The market's dynamics are sensitive to the government's policy stances and other important events related to the cryptocurrencies that may influence market sentiment. Along the historical path of Bitcoin prices, there are two waves of large boom-bust cycles: the 2013–2014 and 2017–2018 episodes. Figure 2 presents the anecdotal relationship between Bitcoin prices and the relevant events of each episode.

The first panel of Fig. 2 shows that the large drop in Bitcoin prices in the middle of April 2013 was triggered by a severe processing delay of transactions. The delay was caused by inefficient capacity at two of the largest exchange intermediaries, Mt. Gox and BitInstant. Afterward, the market recovered and started to surge in October 2013. What followed is that the Bitcoin price reached its historical high in early December 2013. However, the price dropped catastrophically right after the announcement made by the People's Bank of China (PBC) regarding tight regulation on the domestic use of Bitcoins.[3] The markets collapsed again in early February 2014, triggered by the crisis at Mt. Gox.[4] In March 2014, the price continued to fall due to a false report regarding a Bitcoin ban in China and uncertainty over whether the Chinese government would seek to prohibit banks from working with digital currency exchanges.[5]

[3] The PBC strictly prohibited the domestic financial institute from using Bitcoins.

[4] On February 7, 2014, Mt. Gox started to halt all Bitcoin withdrawals due to severe technical issues. The suspension of Bitcoin withdrawals at Mt. Gox then caused a market price collapse. On 28 February, Mt. Gox filed for bankruptcy protection in Tokyo. The company said it had lost almost 750,000 of its customers' Bitcoins and approximately 100,000 of its own Bitcoins, totaling around 7% of all bitcoins. Source: Wikipedia.

[5] On 21 March 2014, Sina (the largest Chinese microblogging site)'s financial live feed issued a now-retracted news report indicating that China's central bank would move to halt all Bitcoin transactions in the



Panel a: The 2013-2014 Episode

Panel b: The 2017-2018 Episode

Fig. 2 Two episodes of Boom-Bust Cycles in the Bitcoin Market. The price series is downloaded from CoinMarketCap.com. The information regarding the events is from Wikipedia: *History of Bitcoin*

The second panel of Fig. 2 illustrates that a series of events regarding the government's tight regulations (or policy stances) triggered several separate rounds of market collapse in 2017 and 2018. For instance, the PBC started to strictly ban the

country effective 15 April. The message was later retracted by the news site following clarification from Chinese regulators. Source: CoinDesk.

initial coin offering (ICO) in September 2017. In January 2018, South Korea asked all the Bitcoin traders to reveal their identities, putting a ban on anonymous trading. Moreover, the negative policy stances against the Cryptocurrency market in March, July and November 2018 triggered recent three rounds of Bitcoin price collapses.[6,7] More intriguingly, in the middle of November 2017, the skyrocketing Bitcoin price in Zimbabwe exchange closely followed domestic political turmoil and an apparent coup.[8] This regional event then sparked a radical reaction in the global market. The price eventually achieved its all-time high (approximately $ 20,000) in December 2017.

The above anecdotal evidence suggests that Bitcoin prices are substantially sensitive to government regulations (or policy stances), as well as to other events that may significantly affect market sentiment. Next, we further manifest these relationships through statistical analysis.

2.2 Policy, sentiment and prices

In this section, we present stylized facts on the market's reaction to government policies and to market sentiment. We provide details on how we measure government policies, sentiment and Bitcoin prices in Appendix A.

Fact 1. *A policy tightening predicts a decrease in future Bitcoin returns and an increase in the probability of a future price collapse.*

We first conduct regressions to document the impact of regulatory policies on the future returns of the Bitcoin market. A return is defined as the period percentage change in Bitcoin prices. In the baseline regressions, we use the return that occurs in the next 15 days as the future return. For regulatory policies, we employ two measures. One is a dummy that indicates whether there is a regulation policy launched on each day. The other one is the number of distinct news events regarding the policies. The results in Table 1 show that (columns 1–4) a tightening policy on the Bitcoin is followed by a future (in the next 15 days) reduction in the Bitcoin price. On the other hand, a loosening policy has no significant impact on the market return. The positive correlation between tightening policy events and the price drops remains robust if we replace the policy tightening dummy with the number of tightening policy events.

Moreover, we document the relationship between the probability of price collapses and regulation policies. A price collapse is defined as a daily drop larger than 10%.

[6] On March 2 2018, the governor of Bank of England Mark Carney said in his speech that cryptocurrencies need to be regulated due to its inherent risks. On July 26 2018, South Korea's financial regulator urged lawmakers to pass the country's first cryptocurrency bill quickly, warning that local exchanges are rife with security flaws and money-laundering risks. On November 14 2018, IMF head Christine Lagarde said that central banks around the world should consider issuing digital currency in order to make digital currency transactions safer. Source of above events: www.bbc.com. Next day, November 15 2018, bitcoin cash officially split into two versions "hard fork".

[7] On May 7 2018, Bill Gates and Warren Buffett lambasted the cryptocurrency market in a TV show at CNBC. They claimed that investing in cryptocurrencies is completely irrational and speculative. Source: www.cnbc.com.

[8] It is highly believed that domestic uncertainty produced a surge in demand for the Bitcoin in Zimbabwe amid a shortage of hard currency.

Table 1 Policy stances and Bitcoin prices

	(1) Return	(2) Return	(3) Return	(4) Return	(5) Collapse	(6) Collapse	(7) Collapse	(8) Collapse
Tight (dummy)	−0.0777**		−0.0772**		0.218**		0.219**	
	(0.0313)		(0.0312)		(0.0964)		(0.0967)	
Loose (dummy)		0.0372	0.0345			0.0869	0.0946	
		(0.0478)	(0.0439)			(0.0981)	(0.0929)	
Tight (#news)				−0.0556**				0.172**
				(0.0281)				(0.0808)
Loose (#news)				0.0382				0.126
				(0.0557)				(0.0851)
Constant	0.0385***	0.0352**	0.0380**	0.0373**	0.227***	0.233***	0.225***	0.226***
	(0.0183)	(0.0159)	(0.0181)	(0.0166)	(0.0384)	(0.0268)	(0.0381)	(0.0384)
Observations	1814	1814	1814	1814	1814	1814	1814	1814
R-squared	0.006	0.001	0.006	0.005	0.009	0.001	0.010	0.010

Tight and Loose are news-based measures of Bitcoin regulation policy stances. We used the EBSCO Newspaper Source to search for news articles regarding Bitcoin or cryptocurrencies and government or taxes or regulations. We then manually identified news articles directly related to Bitcoin regulation policies for policy events and assign Tighten, Loosen or Neutral as the policy stance. Tight (#news) equals the number of Tightening policy events on major newspapers on each day, and Tight (dummy) equals one if there are any Tightening events; the Loose measures are similarly defined. Return is the percentage change of Bitcoin prices in the next 15 days. Collapse is a dummy that equals one if, in the next 15 days, there is at least one day with a daily drop larger than 10%

Newey-West standard errors appear in parentheses, $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$

Columns 5–8 in Table 1 show that a tightening policy significantly raises the probability of a price collapse in the next 15 days. On the other hand, a loosening policy does not have prediction power. This finding suggests that a tightening policy may trigger a market collapse. The result remains robust when alternative indicators for the policy (number of policy events) and the future return of Bitcoins are used.

Fact 2. *Market sentiment positively comoves with Bitcoin prices.*

Market sentiment matters for the Bitcoin prices dynamics as well. To document this relationship, we regress current and future Bitcoin prices on index values of market sentiment. The sentiment index is measured by Twitter/Stocktwits percentage of negative tweets. We use data from Decryptz to measure social media sentiment. Decryptz provides counts of positive, negative and neutral social media messages across Twitter and Stocktwits platforms starting from 2014/09.[9] Our negative sentiment measure equals the bimonthly number of negative messages divided by the number of positive and negative messages. Table 2 reports the empirical results. It can be seen that policy tightening significantly correlates with negative market sentiment, and that, in turn, negative market sentiment strongly predicts lower Bitcoin prices in the next two weeks and strongly correlates with higher chances of Bitcoin price collapses. The results are robust for alternative period definitions. Appendix A provides more robustness analysis.

3 Model

In this section, we construct a model for bubbly Bitcoins to make sense of the empirical relationship between sentiment (or policy stance) and the Bitcoin market. We will also conduct a quantitative exercise to evaluate the aggregate impacts of the Bitcoin market.

The economy is populated by a representative household that owns the firms and supplies labor to the production sector. The economy has two sectors: the real sector and the cryptocurrency (or Bitcoin) sector.[10] The firms in the real sector are facing idiosyncratic investment distortion shocks that follow a continuous distribution.[11] They hire labor and accumulate physical capital to produce consumption and investment goods. The producers (miners) in the cryptocurrency sector utilize real resources to produce cryptocurrencies through a contest based on individual computational power. We first describe the problem of firms in the real sector.

[9] Decryptz (www.decryptz.com) is the Cryptocurrency analysis platform of PsychSignal, a leading social data and sentiment analysis company.

[10] Throughout the paper, we use the terminologies Bitcoin, cryptocurrency and bubble interchangeably.

[11] Investment distortion shocks with a continuous distribution is crucial for the motive of hoarding liquidity. Besides, the firm-level heterogeneity can generate the endogenous trading of cryptocurrency among different investors. This setup also allows us to analyze the channel of resource allocation among individuals through which Bitcoin can affect the economy.

Table 2 Policy stance, sentiment and Bitcoin prices

	(1) Negative Sentiment	(2) BTC price (t-1)	(3) BTC price (t)	(4) BTC price (t+1)	(5) Collapse (t-1)	(6) Collapse (t)	(7) Collapse (t+1)
Tight (dummy)	0.0151 *						
	(0.0089)						
Negative sentiment		0.393	− 1.067	− 1.690**	1.076**	3.648***	0.499
		(0.662)	(0.831)	(0.804)	(0.432)	(0.818)	(1.437)
Constant	− 0.0027	0.0028	9.66E−11	− 0.0081	0.193***	0.193***	0.195***
	(0.0044)	(0.0603)	(0.0414)	(0.0574)	(0.0409)	(0.0385)	(0.0410)
Observations	88	87	88	87	88	88	87
R-square	0.021	0.003	0.023	0.061	0.012	0.140	0.003

Regressions are at the bimonthly level. Negative Sentiment equals the period count of negative messages regarding Bitcoin across Twitter and Stocktwits divided by the period count of positive and negative messages, HP-filtered to remove the trend component. In Column (1), we regress negative sentiment on Tight (dummy), a news-based policy stance dummy that equals one if there is a tightening policy event (but no loosening event) in the current period. In Columns (2)–(4), we regress the Bitcoin price on negative sentiment. The dependent variables are average Bitcoin prices in the last period, the current period and the next period, detrended using the same HP-filter. In Columns (5)–(7), we regress a dummy for the Bitcoin price collapse on negative sentiment. The dependent variables ("Collapse dummy") indicate whether there is a daily drop in the Bitcoin price larger than 10% in the last period, the current period and the next period

Newey-West standard errors appear in parentheses, $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$

3.1 Real sector

The real sector consists of a continuum of heterogeneous firms with one unit of measure. Each firm is indexed by $j \in [0, 1]$. The firm hires labor N_{jt} at the wage rate W_t and accumulates capital K_{jt} to produce final goods according to the Cobb-Douglas production function $A_t K_{jt}^{\alpha} N_{jt}^{1-\alpha}$, where $\alpha \in (0, 1)$ denotes the capital share, A_t is the aggregate technology. We assume that there is no aggregate uncertainty about A_t. The optimal labor is obtained by solving the optimization problem $\Pi_{jt} = \max_{N_{jt} \geq 0} \left\{ A_t K_{jt}^{\alpha} N_{jt}^{1-\alpha} - W_t N_{jt} \right\}$. The optimal decision yields the labor demand as

$$N_{jt} = \left(\frac{1-\alpha}{W_t} A_t \right)^{\frac{1}{\alpha}} K_{jt}. \tag{1}$$

The firm's profit Π_{jt} can be further simplified as $\Pi_{jt} = R_t K_{jt}$, where the marginal rate of return to capital is given by

$$R_t = \alpha A_t^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{W_t} \right)^{\frac{1-\alpha}{\alpha}}. \tag{2}$$

Stochastic bubble We first describe the bubbly cryptocurrency, which is subject to a stochastic burst. We denote χ_t as a binary random variable with $\chi_t = 1$ as the bubbly state where cryptocurrencies exist and $\chi_t = 0$ as the bubbleless state in absence of cryptocurrencies. Following the literature on stochastic bubbles (Kocherlakota 2009), the probability transition matrix for the Markov process of χ_t is given by

$$\Pr(\chi_{t+1} = 0 | \chi_t = 0) = 1, \tag{3}$$

$$\Pr(\chi_{t+1} = 0 | \chi_t = 1) = \pi_{t+1}, \tag{4}$$

where the probability of a bubble burst in $t+1$, $\pi_{t+1} \in [0, 1]$, is observed at time t and assumed to be time-varying. Under the above setup, once the bubble burst, it will never come back. So, the bubbleless state is an absorbing state. To make the analysis nontrivial, we assume $\chi_0 = 1$; i.e., the economy initially stays at the bubbly equilibrium.[12]

In each period, given the stock of cryptocurrency in hand, B_{jt}, the firm j invests I_{jt} of physical capital and $B_{jt+1} - (1-\delta_b) B_{jt}$ of new cryptocurrencies at price P_t, where $\delta_b \in (0, 1)$ is the depreciation rate of the cryptocurrency.[13]

Following Miao et al. (2015a), we introduce an idiosyncratic investment distortion shock, τ_{jt}, on the price of real investments. The firm observes its own τ_{jt} before making its decisions. For tractability, we assume that τ_{jt} is i.i.d. across firms and over

[12] We assume that the bubbly equilibrium can be supported at $t = 0$. We characterize the conditions under which the bubbly equilibrium can be sustained in Sect. 4.

[13] The depreciation rate δ_b captures the fact that a percentage of existing Bitcoins is eternally inactive due to the loss of the private key. Technically speaking, the depreciate rate δ_b is necessary to make the stock of cryptocurrency stationary.

time with a continuous CDF $\mathbf{F}\left(\tau_j;\sigma\right)$ on the support $[\tau_{\min}, \tau_{\max}]$, where σ is the dispersion (or standard deviation) of τ_{jt}. Note that $\tau_{jt} > 1$ represents capital market distortions, e.g., transaction costs or taxation, whereas $\tau_{jt} < 1$ represents as a subsidy to investment,e.g., investment tax credit. The setup of τ_{jt} captures the fact that investment distortions vary across firms/assets and over time.[14] A larger σ implies a larger idiosyncratic uncertainty. In the later analysis, we will show that the idiosyncratic uncertainty provides a crucial motive for holding cryptocurrency. Besides, introducing a continuum of heterogeneous firms through idiosyncratic investment distortion shocks allows us to study the channel of resource misallocation through which the cryptocurrency may affect the real economy.

The firm's dividend D_{jt} is then given by

$$D_{jt} = R_t K_{jt} - \tau_{jt} I_{jt} - \chi_t P_t \left[B_{j,t+1} - (1-\delta_b)\, B_{jt}\right]. \tag{5}$$

We assume that the investment decision is made after the realization of an idiosyncratic shock τ_{jt}. The law of motion of firm j's capital K_{jt} is given by

$$K_{jt+1} = (1-\delta)\, K_{jt} + I_{jt}, \tag{6}$$

where $\delta \in (0, 1)$ is the depreciation rate of physical capital. Equation (6) implies that the effective investment efficiency is $1/\tau_{jt}$.

A necessary condition to support rational bubbles is that economic agents are subject to borrowing constraints (see Miao and Wang 2018). Therefore, we introduce a constraint on equity financing such that

$$D_{jt} \geq 0. \tag{7}$$

Moreover, investment is irreversible, i.e.,[15]

$$I_{jt} \geq 0. \tag{8}$$

Finally, the cryptocurrency holdings are assumed to satisfy no short-sale condition

$$B_{jt+1} \geq 0. \tag{9}$$

Given the individual states $\left\{K_{jt}, B_{jt}, \tau_{jt}\right\}$, let $V_{bt}\left(K_{jt}, B_{jt}, \tau_{jt}\right)$ and $V_{ft}\left(K_{jt}, B_{jt}, \tau_{jt}\right)$ denote the value functions for the bubbly and the bubbleless (or fundamental)

[14] For instance, Goolsbee (1998) provides extensive empirical evidence that investment tax credit varies by assets for many years. Motor vehicles and aircraft, for example, normally have lower rates of credit. Moreover, Song and Wu (2015) demonstrate how idiosyncratic distortions on the price of investment goods can generate capital misallocation and may reduce aggregate TFP in China by 20 percent, even in the late 2000s.

[15] A more generalized setup for equity friction and irreversibility in equations (7) and (8) is $D_{jt} \geq -\zeta_d K_{jt}$ and $I_{jt} \geq -\zeta_i K_{jt}$. See Miao et al. (2015b) and Wang and Wen (2012a) respectively for details. The qualitative results are well preserved.

equilibria, respectively. We use the small letters b and f to label these two equilibria. The recursive optimization problem of firm j in the bubbly and the bubbleless equilibria are given by

$$V_{bt}\left(K_{jt}, B_{jt}, \tau_{jt}\right) = \max\left\{D_{jt} + \frac{\beta\Lambda_{t+1}}{\Lambda_t}\left[(1-\pi_{t+1})\overline{V}_{bt+1}\left(K_{jt+1}, B_{jt+1}\right) + \pi_{t+1}\bar{V}_{ft+1}\left(K_{jt+1}, B_{jt+1}\right)\right]\right\}, \tag{10}$$

and

$$V_{ft}\left(K_{jt}, B_{jt}, \tau_{jt}\right) = \max\left\{D_{jt} + \frac{\beta\Lambda_{t+1}}{\Lambda_t}\bar{V}_{ft+1}\left(K_{jt+1}, B_{jt+1}\right)\right\}, \tag{11}$$

subject to constraints (5) to (9), and

$$\overline{V}_{\kappa t+1}\left(K_{jt+1}, B_{jt+1}\right) \equiv \int_{\tau_{\min}}^{\tau_{\max}} V_{\kappa t+1}\left(K_{jt+1}, B_{jt+1}, \tau_{jt+1}\right) d\mathbf{F}\left(\tau_{jt+1};\sigma\right), \kappa \in \{b, f\}, \tag{12}$$

where $\beta\Lambda_{t+1}/\Lambda_t$ denotes the firm's discount factor, which is derived from the household side.[16]

3.2 Cryptocurrency sector

The cryptocurrency sector produces new cryptocurrencies through so-called mining activities. The miners in this sector compete with others to win a prize by investing in computational power. The victory is probabilistic depending on the miner's own and others' computational power. The reward provided to the winner is paid using the new cryptocurrency, i.e., the Bitcoin. This mechanism provides a channel to create new cryptocurrencies.

Following Dimitri (2017), we assume that there are M_t miners in each period who live only one period.[17] Our main results do not change when the miners can live infinite periods. Each miner is indexed by $i = 1, 2, ..., M_t$ and chooses the investment in computational power H_{it}. The mining reward for solving the puzzle is $Q > 0$ units of the new cryptocurrency. Here, for simplicity, we do not consider the transaction fee in the block that the miner validated as a reward for mining.[18] Let T_{it} denote the waiting time of miner i for solving the puzzle, which is assumed to follow i.i.d. exponential

[16] Since firms are owned by a representative household, as shown later in Sect. 3.3, Λ_t is the household's marginal utility of consumption at period t.

[17] Dimitri (2017) does not consider endogenous entry behavior, so the number of miners is fixed.

[18] In the case of Bitcoin transactions, the reward for miners consists of two things: the newly mined coins in the mining process and the transaction fees in the block that the miner validated. According to the Bitcoin protocol, the amount of transaction fee for each transaction is fixed, up to a couple of US dollar cents regardless of the amount of Bitcoins.

distribution with parameter H_{it}/d_t, where d_t is an indicator of the difficulty for solving the puzzle and is adjusted by the Bitcoin protocol.

The miner employs the final good to construct the computational power. For analytical convenience, we assume that the computational power is transformed according to a linear technology, $H_{it} = X_{it}/a$, where $1/a$ denotes the transformation efficiency. Therefore, the total cost (measured by the final good) for investing in H_{it} computational power is simply aH_{it}. The miner can win the competition and obtain the reward only if he solves the puzzle in the shortest time. Therefore, the probability that miner i can be rewarded is $H_{it}/\sum_{k=1}^{M_t} H_{kt}$. Online Appendix S.3 provides more detailed proof. The profit function for miner i can be expressed as

$$\Pi_{it}^c = \begin{cases} P_t Q - aH_{it}, & \text{with prob. } H_{it}/\sum\limits_{k=1}^{M_t} H_{kt} \\ -aH_{it}, & \text{with prob. } 1 - H_{it}/\sum\limits_{k=1}^{M_t} H_{kt} \end{cases}. \tag{13}$$

Given the others' investments in computational power, miner i chooses his own H_{it} to maximize the expected profit $\mathbb{E}_t \Pi_{it}^c = (1-\pi_t)\left(H_{it}/\sum_{k=1}^{M_t} H_{kt}\right) P_t Q - aH_{it}$. The miner's expected profit takes into account the risk of a bubble burst, π_t, since the cryptocurrency bubble may burst after the miners make their investment decisions.[19] The probability of the survival of the bubble is $1-\pi_t$. The optimal H_{it} then satisfies

$$(1-\pi_t)\, P_t Q \sum_{k \neq i}^{M_t} H_{kt} \Big/ \left(\sum_{k=1}^{M_t} H_{kt}\right)^2 = a. \tag{14}$$

The last equation characterizes the optimal reaction function for the individual miner $i = 1, 2, ..., M_t$. The Nash equilibrium is defined as the solution of the equation system constituted by the optimal reactions. It is straightforward to show that each miner invests the same amount of computational power, which satisfies

$$H_{it} \equiv H_t^* = \frac{M_t - 1}{M_t^2 a} (1-\pi_t)\, P_t Q. \tag{15}$$

Then, the expected profit for the individual miner is $\mathbb{E}_t \Pi_{it}^c = (1-\pi_t)\, P_t Q/M_t^2$. Since the input satisfies $X_{it} = aH_{it}$, after aggregation, the total input used for the cryptocurrency sector is $X_t = aM_t H_t^*$.

To determine the number of miners, we assume that the potential entrants can enter the cryptocurrency sector by paying a fixed entry cost, $\nu > 0$, in terms of a final good.[20] We assume that the entry cost is financed from the household side. The free entry condition is given by $\nu = (1-\pi_t)\, P_t Q/M_t^2$, which implies that the total

[19] This setup of timing may fit the reality better, since investing in Bitcoin mining may take time and involve fixed cost (e.g., GPUs and warehouses). Alternatively, we can assume that the miners' investment decisions are made after observing the state of the bubble. The main analysis remains valid.

[20] The fixed entry cost, in reality, corresponds to the working capital of a miner such as a computer.

number of miners in the equilibrium is $M_t = [(1-\pi_t) P_t Q/\nu]^{\frac{1}{2}}$. A boom in the price of the cryptocurrency is expected to increase the number of miners. The reduced form relationship of the real resource used for producing Bitcoins and the price (X_t, P_t) can be further expressed as

$$X_t = \left(1 - \frac{1}{M_t}\right)(1-\pi_t) P_t Q = \left\{1 - \left[\frac{(1-\pi_t) P_t Q}{\nu}\right]^{-\frac{1}{2}}\right\}(1-\pi_t) P_t Q. \tag{16}$$

The last equation indicates that the total input used for producing cryptocurrencies strictly increases with P_t, the price of cryptocurrency.[21]

3.3 Household

We introduce a representative household to close the model. Due to concerns about neatness, the household's labor supply is fixed to be one. As in Wang and Wen (2012b) and Miao et al. (2015a), the household chooses consumption, C_t, and stock holdings for each firm j in the real sector, s_{jt+1}, to maximize expected lifetime utility $\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$. The budget constraint is given by

$$C_t + \int_0^1 s_{jt+1}\left(V_{jt} - D_{jt}\right) dj + M_t \nu = W_t + \int_0^1 s_{jt} V_{jt} dj + \Pi_t^c - T_t, \tag{17}$$

where D_{jt} denotes the dividend distributed from the real sector, $M_t \nu$ is the total expenditures used to finance the entry cost of new miners, Π_t^c is the profit distributed from the cryptocurrency sector, and T_t represents lump-sum taxes satisfying $T_t = \int_0^1 \left(1-\tau_{jt}\right) I_{jt} dj$. Note that we introduce this lump-sum tax, T_t, to offset the impact of investment distortions, τ_{jt}, on the household side.

Let Λ_t denote the Lagrangian multiplier for the budget constraint (17). The optimal consumption decision implies $u'(C_t) = \Lambda_t$. The optimal condition for the equity holding of firm j, s_{jt+1}, is given by

$$V_{jt} = D_{jt} + \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{jt+1}, \tag{18}$$

where $\beta \Lambda_{t+1}/\Lambda_t$ denotes the discount factor. The last equation essentially describes the recursive process of the firm's value function in the real sector.[22]

[21] We need the condition $(1-\pi) P_t Q > \upsilon$ to ensure a positive X_t. To see this, (16) indicates that the resource used for producing Bitcoins is $X_t = \left\{1 - \left[\frac{(1-\pi_t) P_t Q}{\upsilon}\right]^{-\frac{1}{2}}\right\}(1-\pi_t) P_t Q$. A positive X_t requires $\frac{(1-\pi_t) P_t Q}{\upsilon} > 1$ or $(1-\pi) P_t Q > \upsilon$. This condition is guaranteed because from the free entry condition. In the quantitative exercise, our calibration ensures $M_t > 1$. Therefore, the condition $(1-\pi_t) P_t Q > \upsilon$ always holds.

[22] Note that the firm value V_{jt} depends upon the state of the equilibrium. That is, $V_{jt} = V_{bt}\left(K_{jt}, B_{jt}, \tau_{jt}\right)$ for the bubbly equilibrium, and $V_{jt} = V_{ft}\left(K_{jt}, B_{jt}, \tau_{jt}\right)$ for the bubbleless equilibrium. Moreover, the expected firm value in the next period $\mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{jt+1}$ follows the expressions in Eqs. (10) and (11).

4 Characterization

4.1 Firm's optimal decisions

We now characterize the decision rules of the individual firms, which are heterogeneous in terms of their idiosyncratic investment shocks τ_{jt}. Therefore, the firm's optimal decisions are state contingent and can be solved by using a guess-and-verify strategy. See Appendix 1 for more details.

Briefly speaking, due to the linear structure of the firm's problem, the optimal investment decision follows a trigger strategy. If the investment shock τ_{jt} is relatively low, i.e., $\tau_{jt} < \tau_{jt}^*$, the firm opts to invest as much physical capital as possible. In this case, the firm will sell the cryptocurrencies in hand and fully utilize internal funds (the current profit Π_t) to finance the real investment. Therefore, both the equity financing constraint (7) and the no short-sale constraint (9) bind. If the investment shock turns out to be relatively high, i.e., $\tau_{jt} > \tau_{jt}^*$, investing in physical capital is not desirable. The firm opts to store its liquidity in the form of cryptocurrencies. Proposition 1 summarizes the individual firm's optimal decisions as well as the evolution of Bitcoin prices.

Proposition 1 *Given the aggregate states, the optimal investment decision for individual firm j with τ_{jt} follows a trigger strategy*

$$I_{jt} \equiv K_{jt+1} - (1-\delta)K_{jt} = \begin{cases} \frac{R_t K_{jt} + \chi_t P_t (1-\delta_b) B_{jt}}{\tau_{jt}}, & \text{if } \tau_{jt} < \tau_t^* \\ 0, & \text{if } \tau_{jt} \geq \tau_t^* \end{cases}, \tag{19}$$

where $\tau_t^* \equiv \chi_t \tau_{bt}^* + (1-\chi_t)\, \tau_{ft}^*$ for $\chi_t \in \{0, 1\}$, and the cutoff values τ_{bt}^* and τ_{ft}^* are jointly determined by

$$\tau_{bt}^* = \frac{\beta \Lambda_{t+1}}{\Lambda_t} (1-\pi_{t+1}) \left[R_{t+1} \Phi\left(\tau_{bt+1}^*; \sigma\right) + \tau_{bt+1}^* (1-\delta) \right] + \pi_{t+1} \tau_{ft}^*, \tag{20}$$

$$\tau_{ft}^* = \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[R_{t+1} \Phi\left(\tau_{ft+1}^*; \sigma\right) + \tau_{ft+1}^* (1-\delta) \right]. \tag{21}$$

The liquidity premium $\Phi(\tau^*; \sigma)$ is defined as

$$\Phi\left(\tau^*; \sigma\right) \equiv \int_{\tau_{\min}}^{\tau_{\max}} \max\left\{ \frac{\tau^*}{\tau}, 1 \right\} d\mathbf{F}(\tau; \sigma). \tag{22}$$

The price of the cryptocurrency is determined by

$$P_t = \left[(1-\pi_{t+1})(1-\delta_b) \frac{\beta \Lambda_{t+1}}{\Lambda_t} P_{t+1} \Phi\left(\tau_{bt+1}^*; \sigma\right) \right]. \tag{23}$$

Proof See Appendix C. □

Equation (19) states that the optimal investment for those firms with lower distortion ($\tau_{jt} < \tau_t^*$) is financed from the firm's internal funds $R_t K_{jt}$ and the cryptocurrency in hand $P_t (1 - \delta_b) B_{jt}$. Moreover, the Euler equation (20) indicates that Tobin's Q consists of two components: if the bubble survives in the next period, one unit of physical capital will generate a value of $R_{t+1} \Phi \left(\tau_{bt+1}^*; \sigma\right) + \tau_{bt+1}^* (1 - \delta)$; however, if the bubble bursts, the value of one unit of installed capital would be τ_{ft}^*. It is worth noting that holding one more unit of physical capital can relax the financial constraint in the next period. Therefore, the liquidity premium term $\Phi(\tau_{\kappa t+1}^*; \sigma)$ enters the Euler equation. This term also captures the option value of holding cryptocurrency. When the investment distortion τ_{jt} is small, i.e., $\tau_{jt} < \tau_{bt}^*$, the cryptocurrency in hand can provide additional liquidity to help the firm relax the financial constraint. As a result, the current price of the cryptocurrency, P_t, is equal to the expected discounted value of the price times the liquidity premium in the next period; see Eq. (23).[23]

4.2 Aggregation and general equilibrium

Let $Z_t = \int Z_{jt} dj$ and $Z \in \{K, B, Y, N\}$ denote the aggregate variables in the real sector. After aggregating at the firm level, the aggregate investment is given by

$$K_{t+1} - (1 - \delta) K_t = [R_t K_t + \chi_t P_t (1 - \delta_b) B_t] \, G\left(\tau_t^*; \sigma\right), \tag{24}$$

where $\tau_t^* \equiv \chi_t \tau_{bt}^* + (1 - \chi_t) \tau_{ft}^*$ and $\chi_t \in \{0, 1\}$ is an indicator for the bubbly equilibrium; $G(\tau^*; \sigma) = \int_{\tau_{\min}}^{\tau^*} \frac{1}{\tau} d\mathbf{F}(\tau; \sigma)$ resembles the aggregate investment efficiency. For analytical convenience, we normalize the mean of $\frac{1}{\tau}$ to be 1.

Due to the CRS production function, the aggregate output Y_t satisfies

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{25}$$

where, by assumption, $N_t = 1$. The goods market clearing condition derives the resource constraint

$$C_t + K_{t+1} - (1 - \delta) K_t + X_t + M_t \nu = Y_t, \tag{26}$$

where X_t is the total input used for producing cryptocurrencies and $M_t \nu$ is the total entry cost for new miners. The clearing condition in the cryptocurrency market gives

$$B_{t+1} = (1 - \delta_b) B_t + \chi_t Q, \tag{27}$$

[23] As the bubble cannot come back after its burst, Tobin's Q in the bubbleless equilibrium, τ_{ft}^*, evolves according to (21). In a frictionless economy, the firm does not suffer from market friction, so there is no additional value in holding the cryptocurrency. In this extreme case, we must have $\tau_{\kappa t}^* = \tau_{\min}$, and the liquidity premium becomes $\Phi\left(\tau^*; \sigma\right) = 1$. As a result, the bubbly equilibrium cannot be supported.

where Q is the amount of new produced cryptocurrency in each period. Finally, the optimal demand for capital and labor implies

$$R_t = \alpha \frac{Y_t}{K_t} \quad \text{and} \quad W_t = (1-\alpha)\frac{Y_t}{N_t}. \tag{28}$$

A general equilibrium is defined as the paths of quantities and prices $\{Y_t\}$, $\{K_{t+1}\}$, $\{B_t\}$, $\{C_t\}$, $\{M_t\}$, $\{X_t\}$, $\{\tau^*_{\kappa,t}\}$, $\{R_t\}$, $\{W_t\}$, $\{P_t\}$ such that households and firms optimize and markets clear. Appendix B summarizes the full dynamic system.

4.3 Steady state

Non-cryptocurrency steady state In the non-cryptocurrency (or bubbleless) steady state, the cryptocurrency does not exist; i.e., $P = B = 0$. We use the subscript f to denote this equilibrium. From the Euler equation (21), we can solve the marginal product of capital in the bubbleless equilibrium, $R_f \equiv \alpha K_f^{\alpha-1}$, as a function of the cutoff τ_f^*

$$R_f = \frac{1-\beta(1-\delta)}{\beta}\tau_f^*\left[\Phi\left(\tau_f^*;\sigma\right)\right]^{-1}. \tag{29}$$

Substituting the last equation into aggregate investment (24) yields an implicit function for τ_f^* (note that $PB = 0$)

$$\frac{\beta\delta}{1-\beta(1-\delta)} = \frac{\tau_f^* G\left(\tau_f^*;\sigma\right)}{\Phi\left(\tau_f^*;\sigma\right)}. \tag{30}$$

where $G(\tau^*;\sigma) = \int_{\tau_{\min}}^{\tau^*}(1/\tau)\,d\mathbf{F}(\tau;\sigma)$. Appendix C proves that a unique bubbleless equilibrium exists under the condition $\int_{\tau_{\min}}^{\tau_f^*}(1/\tau)\,d\mathbf{F}(\tau;\sigma) < 1/\alpha$.

Cryptocurrency steady state From the price equation (23), the liquidity premium can be solved as

$$\Phi\left(\tau^*;\sigma\right) = \int_{\tau_{\min}}^{\tau_{\max}} \max\left\{\frac{\tau_b^*}{\tau}, 1\right\} d\mathbf{F}(\tau;\sigma) = \frac{1}{(1-\pi)(1-\delta_b)\beta}, \tag{31}$$

which is increasing in the probability of bubble burst π. Intuitively, a higher risk of a burst implies that to make investors hold the bubble, they must be compensated by a higher liquidity premium.

It is straightforward to verify that a unique solution for τ_b^* exists if and only if $\tau_{\max} > \frac{1}{(1-\pi)(1-\delta_b)\beta}$. Thus τ_b^* increases with the probability of burst π but decreases with the dispersion σ. That is, when there is higher risk in the cryptocurrency market (π is smaller) or less severe investment distortion (σ is smaller), more firms tend to invest in the real sector.

Given that τ_b^* exists, according to the aggregate investment equation (24) and the evolution of physical capital (6), the market value of cryptocurrency PB satisfies

$$PB = \Delta \times K_b, \tag{32}$$

where $\Delta = \left\{\delta \left[G\left(\tau_b^*; \sigma\right)\right]^{-1} - R_b\right\} / \left(1 - \delta_b\right)$ and R_b is the marginal product of capital in the bubbly steady state. Therefore, the existence of a bubbly equilibrium, i.e., $P > 0$, is equivalent to the condition $\Delta > 0$. The economic interpretation for Δ is intuitive. The term $G\left(\tau_b^*; \sigma\right)$ in Δ captures the efficiency for investing in physical capital. The term $\delta \left[G\left(\tau_b^*; \sigma\right)\right]^{-1}$ indicates the amount of investment that needs to be financed in the steady state (if we normalize capital to be 1). R_b is the amount of internal funds that the firm earns from the production. The gap between these two terms is the additional liquidity that needs to be financed through the cryptocurrency. Low efficiency (i.e., $G\left(\tau_b^*; \sigma\right)$ is small) implies that firms, ceteris paribus, demand a larger amount of external liquidity to finance their investments. As a result, the aggregate bubble-to-capital ratio Δ is higher.

Assumption 1 The upper bound of idiosyncratic investment distortion $\tau_{\max}$ is sufficiently large, satisfying $\tau_{\max} > \frac{1}{(1-\pi)(1-\delta_b)\beta}$; $G\left(\tau^*; \sigma\right)$ strictly decreases with the dispersion of idiosyncratic investment distortion shock (σ) for any $\tau^* \in [\tau_{\min}, \tau_{\max}]$.

The first part of the assumption guarantees the existence of the cutoff τ_b^* in the bubbly equilibrium. The second part of the assumption indicates that given τ^* is fixed, an increase in the dispersion of idiosyncratic investment distortion, σ, would reduce the aggregate investment efficiency $G\left(\tau^*; \sigma\right)$.

Proposition 2 *Under Assumption 1, a unique bubbly equilibrium can be supported if and only if the probability of a burst satisfies $\pi < \bar{\pi}\left(\sigma\right)$, where $\bar{\pi}\left(\sigma\right)$ is implicitly determined by*

$$\frac{\beta\delta}{1 - \beta\left(1 - \delta\right)} \frac{1}{\left(1 - \bar{\pi}\right)\left(1 - \delta_b\right)\beta} = \tau^* G\left(\tau^*; \sigma\right). \tag{33}$$

and τ^* satisfies

$$\Phi\left(\tau^*; \sigma\right) = \frac{1}{\left(1 - \bar{\pi}\right)\left(1 - \delta_b\right)\beta}. \tag{34}$$

Furthermore, the upper bound $\bar{\pi}\left(\sigma\right)$ increases with σ.

Proof See Appendix C. □

The proposition illustrates that the existence of the bubbly cryptocurrency requires that the probability of a bubble burst, π, cannot be large; i.e., the sentiment cannot be overly pessimistic. This is straightforward to understand. A high risk of a bubble burst would reduce the incentives for the firms to purchase the cryptocurrency. As a result, to guarantee a positive price of the bubble, sentiment $(\frac{1}{\pi})$ must be high.

The proposition also suggests that the lower limit of sentiment negatively depends on the dispersion of idiosyncratic investment distortion σ. The intuition is that a large dispersion of an idiosyncratic investment distortion shock reduces aggregate investment efficiency. Therefore, firms demand more cryptocurrencies as liquid assets to finance their investments. This relaxes the condition for the existence of a bubbly equilibrium (i.e., $\frac{1}{\pi}$ decreases).[24]

5 Quantitative exercise

We now use our production economy model with bubbly Bitcoins to conduct a quantitative analysis of (1) the conditions for the existence of Bitcoins, (2) the steady-state impact of sentiment shocks on Bitcoins and the real economy, (3) the dynamic impact of sentiment shocks and the nature of a dynamic comovement in the Bitcoin economy and lastly, (4) the consequences of a Bitcoin bubble burst on real investment and on social welfare.

We first specify the parameter values as follows. One period in our model corresponds to one quarter. For the standard parameters, the calibrations just follow the related literature. In particular, the discount rate β is set to 0.98. The depreciation rate of physical capital δ is set to 0.025. The capital share in the production function of the real sector α is set to 0.4. The depreciation rate of the cryptocurrency δ_b is set to 0.1.[25] The quantity of new cryptocurrency Q is normalized to be 1. Regarding the efficiency parameter in cryptocurrency production a, as it does not alter the model dynamics, we normalize it to be 1. For the fixed entry cost, we set it to be 0.001.[26] We assume that effective investment efficiency $\varepsilon = 1/\tau$ follows a Pareto distribution with CDF $1-(\varepsilon/\varepsilon_{\min})^{-\eta}$. We further normalize the mean of ε to be 1, i.e., $\mathbb{E}(\varepsilon) = 1$, so we must have $\varepsilon_{\min} = 1/\tau_{\max} = 1 - 1/\eta$. The shape parameter η indicates the dispersion of the idiosyncratic investment distortion shock, i.e., $\sigma = \sqrt{\frac{1}{\eta(\eta-2)}}$, so a smaller η implies a larger dispersion of investment distortion. The probability π reflects the riskiness of holding cryptocurrencies. The parameters σ and π are crucial for the existence of a bubbly equilibrium. Therefore, in our analysis, we consider different combinations of values of these two parameters.

[24] It is worth noting that the existence condition for the bubbly equilibrium is irrelevant to the mining sector. This is because given the stock of real capital K_b, the term $\Delta\left(\tau_b^*\right)$ shifts the demand curve of Bitcoin; see Eq. (32). Since the supply of Bitcoin is fixed at the amount of $\frac{Q}{\delta_b}$, the equilibrium price P is solely determined by the demand side.

[25] The depreciate rate of cryptocurrencies may reflect the rate of dormant coins due to the loss of the private key or more generally, the exit rate of cryptocurrencies. Since there are no reliable data that can be used to accurately calibrate the value of δ_b, we conservatively set δ_b to 0.1, implying that 40% of the cryptocurrencies per year are inactive. The main predictions in our model remain robust for the values of δ_b.

[26] This parameter should be small enough to make the equilibrium number of the miners greater than 1. The model's dynamics do not rely on the value of ν.

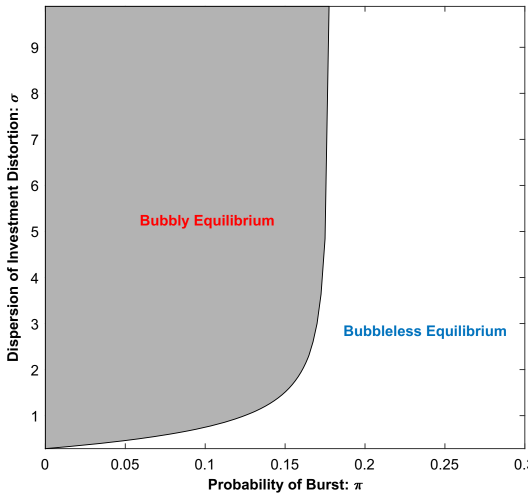


Fig. 3 Feasible set for the bubbly equilibrium

5.1 Bubbly steady state

We first discuss the feasible set of the dispersion of the idiosyncratic investment distortion σ and the burst probability π for the existence of a bubbly equilibrium. Figure 3 plots the feasible combinations of values of σ and π that guarantee the existence of cryptocurrencies. The envelope of the feasible set shows that the value for the probability of a burst, π, and the extent of investment distortion, σ, have a positive relationship. This result confirms the argument in Proposition 2. That is, to support the bubbly equilibrium, the higher probability of a burst requires a more severe distortion. The reason for this is quite intuitive. The incentive for firms to hold cryptocurrencies is mainly due to the fact that the cryptocurrencies can provide liquidity. When the distortion is larger, i.e., σ is larger, the demand for liquid assets is stronger. As a result, the existence condition for the cryptocurrency equilibrium becomes looser.

Impact of sentiment To further document the impact of sentiment on the steady-state values for the aggregate economy, in our baseline analysis we specify the dispersion of investment distortion σ to be 0.5774, which corresponds a shape parameter $\eta = 3$. We can show that the steady state in the cryptocurrency equilibrium implies that Tobin's Q, τ_b^*, is strictly increasing in π. However, the capital return R_b is nonmonotonic in π (see Equation C.15 in Appendix C). Figure 4 shows that, under a moderate distortion, the value of the cryptocurrency relative to physical capital PB/K is decreasing in π. That is, when market sentiment becomes more negative ($\frac{1}{\pi}$ decreases), the relative importance of the cryptocurrency is reduced. The figure also shows that the price of the cryptocurrency is also decreasing in π, implying that a negative shift in market sentiment in the model (a decrease in $\frac{1}{\pi}$) causes a depression in the Bitcoin price. The pattern is consistent with the empirical facts discussed in the previous analysis.

Moreover, Fig. 4 shows that sentiment has nonmonotonic impacts on the real investment and real output. We illustrate the mechanism by the decomposing the aggregate

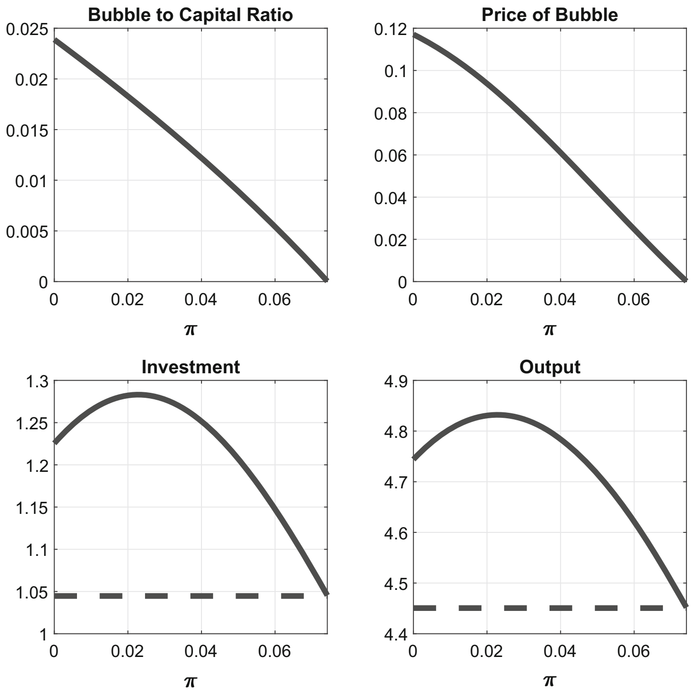


Fig. 4 Bubbly steady state under different sentiment. We set the shape parameter of Pareto distribution to be 3, implying the dispersion of investment distortion $\sigma = 0.5774$. The other parameters, except π, are evaluated at their calibrated values. The dashed lines in the bottom panels indicate the steady-state values in a Bubbleless equilibrium

investment in the bubbly equilibrium as defined in Eq. (24). In particular, the first term $R_t K_t + (1 - \delta_b) P_t B_t$ indicates the liquidity required for investment, which consists of internal fund $R_t K_t$ and external fund $(1 - \delta_b) P_t B_t$. The second term $G\left(\tau_t^*; \sigma\right)$ indicates the investment efficiency. A change in sentiment has opposite effects on the liquidity and the efficiency components, leading to a non-monotonic relationship between the sentiment π and the aggregate investment. In particular, a more negative market sentiment (π increases) reduces the demand for cryptocurrency, which further mitigates the resource misallocation caused by the cryptocurrency sector, and therefore improves the allocation efficiency, i.e., $G\left(\tau_t^*; \sigma\right)$ increases. This indicates the efficiency-improving channel. On the other hand, a more negative sentiment reduces the liquidity for financing the investment due to the depressed cryptocurrency prices, i.e., $(1 - \delta_b) P_t B_t$ declines. This indicates the liquidity-reducing channel. The overall effects depend on which channel dominates.When the market sentiment is pessimistic (π is large), the liquidity-reducing channel dominates since the liquidity constraints are tighter. When the market sentiment is optimistic (π is low), the efficiency-improving channel dominates since the large cryptocurrency sector causes more severe misallocation.[27]

[27] Figure 4 also confirms the argument in Corollary S.1.1 that the bubbly steady state has a lower cutoff τ_b but higher output and investment. In particular, the two bottom panels in Fig. 4 show that the output

5.2 Dynamics

Impact of a sentiment shock We now investigate the dynamic consequences of a sentiment shock on the cryptocurrency market and the real economy. Figure 5 shows that when the level of steady state sentiment is low (e.g., $\pi = 0.02$), a negative sentiment shock (π_t increases) would depress the bubble price but stimulate the real economy. When the steady-state sentiment is relatively high ($\pi = 0.04$), a negative sentiment shock again dampens the bubble price. However, in this case, the negative sentiment shock also causes an economic recession. The underlying mechanism is similar to that in the previous comparative static analysis. For a low steady-state value of π (e.g., $\pi = 0.02$), a negative sentiment shock raises the aggregate investment because the efficiency-improving channel dominates. Whereas, for a high steady-sate value of π (e.g., $\pi = 0.04$), a negative sentiment shock dampens the aggregate investment because the liquidity-reducing channel dominates. The online appendix provides more quantitative results for the dynamics of these two channels under a negative sentiment shock.

The relationship between the cryptocurrency market and the real economy predicted by our model, indeed, is well supported by the real data. In particular, Appendix D documents that for the U.S. economy, the price of cryptocurrencies presents a strong counter-cyclical pattern. For the Chinese economy, the cryptocurrency market positively comoves with the real economy. Therefore, the above quantitative exercises indicate that our model is able to explain the observed patterns in the real economy.[28]

Consequences of a bubble burst It is important to evaluate the possible consequences caused by a collapse of the cryptocurrency market. To do so, we consider an extreme case: the economy initially stays at the bubbly equilibrium; then, the cryptocurrency bubble bursts; i.e., the price P_t collapses to zero. Figure 6 displays the transition dynamics and shows that the burst of the bubble causes a contraction in output and investment.[29] The magnitude of the contractionary effect is amplified when the investment distortion becomes larger (or σ becomes larger). This is because a larger investment distortion increases the size of the bubble; therefore, the burst of the bubble may lead to a deeper recession. As the figure shows, when the investment distortion is small (e.g., $\sigma = 0.3536$), the market collapse merely causes a small contraction in the real economy (solid lines). However, for the case with a larger distortion (e.g., $\sigma = 0.5774$), the burst of the bubble triggers a much prolonged recession (dashed lines). Despite the adverse impact on output and investment, the collapse of the cryptocurrency market prompts consumption along the transition path. As a result, social welfare is improved. This result echoes the welfare implication under a nega-

Footnote 27 continued
and investment are uniformly above those in the bubbleless equilibrium (the dashed lines). Therefore, using the cryptocurrencies as liquid assets stimulates the firms' investments and productions.

[28] One possible explanation is that the U.S. market holds a relatively optimistic belief on the Bitcoin bubble (π is low), whereas the Chinese investors hold a relatively pessimistic belief (π is high).

[29] In Online Appendix S.2, we also investigate the impact of the emerging Bitcoin sector on the aggregate economy. The quantitative exercise shows that the output increases but not that much after the emergence of a Bitcoin sector, because producing Bitcoin crowds out real resources allocated to the real sector.

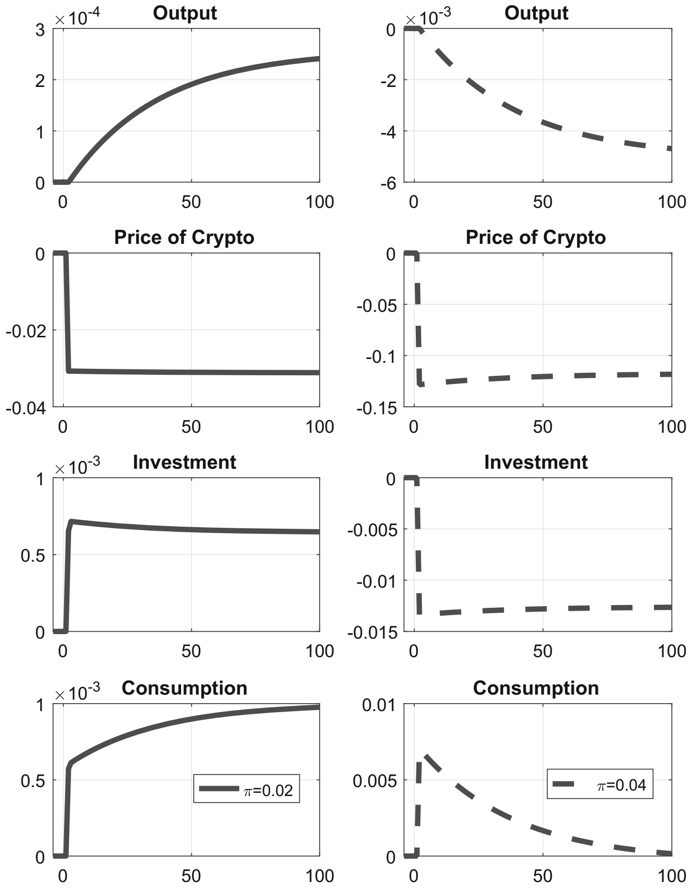


Fig. 5 Transition dynamics under a sentiment shock. We assume that in the initial period, the economy stays at the steady state. In the first period, the probability of burst π permanently increases by 10%. The transition dynamics are the percentage deviation from the initial steady state. All the parameters except π are evaluated at their calibrated values. We set the shape parameter of Pareto distribution to be 3, implying the dispersion of investment distortion $\sigma = 0.5774$

tive sentiment shock, where the expansion of the cryptocurrency sector reduces social welfare.

Figure 7 presents the welfare implication of the bubble burst under different magnitudes of investment distortion σ (the left panel). In particular, the measurement of welfare, λ, is defined as

$$\sum_{t=1}^{\infty} \beta^t u\left(C_t\right) = \frac{1}{1-\beta} u\left(\left(1+\lambda\right) C_b\right), \tag{35}$$

where C_b is the steady-state consumption in the initial period, and $\{C_t\}_{t=1}^{\infty}$ is the sequence of consumption on the transition path after the bubble burst. The variable λ

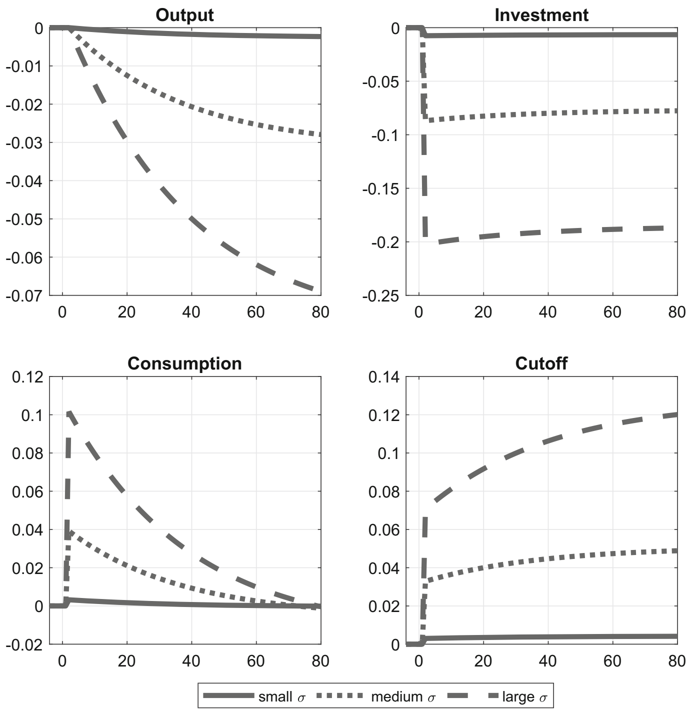


Fig. 6 Consequences of a market collapse. We assume that in the initial period, the economy stays at the bubbly steady state. In the first period, the cryptocurrency market collapses; i.e., $P_t = 0$. The transition dynamics are the percentage deviation from the initial bubbly steady state. We set the probability of the bubble burst, π, to be 0.02. The other parameters are evaluated at their calibrated values. The small, medium and large σ correspond to $\sigma = \{0.3536, 0.5774, 1.5076\}$, respectively

measures the percentage of consumption that must be compensated to the households such that they are indifferent about the initial steady state and the state after the bubble burst. From Fig. 7, it can be seen that the magnitude of welfare improvement relies on either the severity of the investment distortion σ (the left panel) or the size of the bubble (the right panel). A larger σ, for instance $\sigma = 0.5774$, implies a larger bubble in the bubbly equilibrium ($PB/Y = 20\%$). In this case, the welfare improvement is more sizeable, approximately 3.6% of consumption. However, for a smaller σ, e.g., $\sigma = 0.3536$ (corresponding to $PB/Y = 0.74\%$), the welfare improvement is fairly small (0.1% of consumption).

6 Conclusion

The Bitcoin markets present enormous volatility and the price dynamics are significantly sensitive to both market sentiment and policy stances. It is difficult to not treat Bitcoins as asset bubbles. Moreover, the Bitcoin market is resource consuming because it requires large-scale mining activities. Not surprisingly, immense public interest and

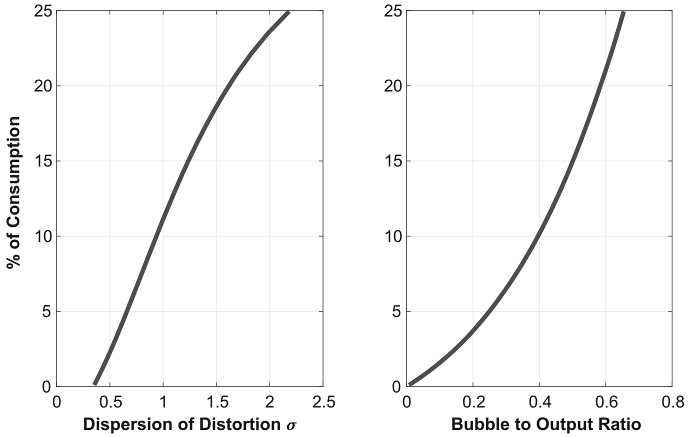


Fig. 7 Welfare implication of a market collapse. For each given value of σ, we compute welfare according to Eq. (35) and the bubble to output ratio ($\frac{PB}{Y}$) in the initial bubbly equilibrium. The left panel plots the relationship between the σ and the welfare along the transition. The right panel plots $\frac{PB}{Y}$ against welfare. We set the probability of a bubble burst, π, to be 0.02. The other parameters are evaluated at their calibrated values

debate has been devoted to Bitcoin-like cryptocurrencies. However, there is little theory proposed in studies that consider the causes and aggregate consequences of Bitcoin bubbles. To this end, our paper constructs a macroeconomic model that considers cryptocurrencies to be risky and costly bubbles. The model is featured with the market structure of Bitcoins and heterogeneous investors who face idiosyncratic investment distortions. The financial market's incompleteness and investment distortions push the investors to hold Bitcoins as liquid assets. Therefore, the intrinsically worthless Bitcoins can emerge as rational bubbles. We characterize market sentiment by modeling the Bitcoin as stochastic bubbles. A higher probability of bursting then corresponds to more pessimistic sentiment. We show that market sentiment plays a key role in driving the fluctuations in the Bitcoin price. In particular, a deterioration in market sentiment unambiguously depresses the Bitcoin price.

The presence of bubbly Bitcoins has competing effects on investment: on the one hand, the Bitcoin facilitates real investment by providing more market liquidity to financially constrained firms, while on the other hand, the associated mining activities are resource consuming and thus crowd out investment. Consequently, the aggregate consequences of sentiment fluctuations are ambiguous. When the sentiment is relatively optimistic (i.e., the probability of a collapse is low), a further deterioration in sentiment may stimulate the aggregate economy. The result turns out to be the opposite when the market sentiment is relatively pessimistic. Therefore, our model is able to demonstrate the diverging cyclical relationship between the Bitcoin market and the aggregate output observed in the real economy, such as in the US and China, the largest developed and developing economies, respectively. We also investigate the implications of the collapse of Bitcoin bubbles. It turns out that the bubble burst is followed by a slump in aggregate output and investment and by an increase in aggregate consumption. That is, through the lens of our framework, the absence of the Bitcoin market, in general, improves social welfare. The quantitative results, however, rely heavily on the severity of the market distortion, which, in turn, determines the size of the Bitcoin bubbles.

Our model considers a real economy. For future research, it will be intriguing to extend this to a monetary model to study the optimal monetary policy in the presence of Bitcoins. Another research line is studying the macro-prudential policy when different Bitcoin-like cryptocurrency bubbles coexist with housing bubbles and stock bubbles. Finally, it is worth noting the way we have tried to endogenize the probability of bubble burst is lack of micro-foundation. It is an important and challenging issue that needs to be dealt with in future work.

Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/s00199-021-01389-y.

Appendix

A Data description and variable construction

Bitcoin price We retrieve the time series of the Bitcoin price from CoinMarketCap.com. This website provides daily opening, high, low, and closing prices of Bitcoin that goes back to April 28, 2013, which is sufficient for this analysis. Since the Bitcoin market is open 24 hours each day, the daily time series uses 00:00:00 in the Greenwich Mean Time to separate the days. We use the daily closing price as our preferred Bitcoin price measure to make sure the Bitcoin return we compute and use in the predictive regressions follows the Bitcoin regulation events we document timing-wise.

Measures of Bitcoin regulation policy stances To investigate the impact of government policy on the value and the risk of Bitcoin, we construct a comprehensive catalog of Bitcoin regulation policy events, which we then use to construct Bitcoin policy stance measures at the daily level.

We use the EBSCO Newspaper Source database to search for news articles regarding Bitcoin or cryptocurrencies and government or taxes or regulations. The EBSCO Newspaper Source database provides cover-to-cover texts of 489 major newspapers in 24 countries including Australia, Canada, France, India, Israel, Japan, Mexico, Saudi Arabia, Thailand, the U.K. and the U.S.[30] We query the EBSCO Newspaper Source database on May 2, 2018, using the following search terms:

```
(bitcoin AND tax) OR (bitcoin AND government)
OR (bitcoin AND regulation) OR (cryptocurrency AND tax)
OR (cryptocurrency AND government) OR (cryptocurrency AND regulation)
```

We then manually read each returned news article in the EBSCO Newspaper Source database, identify those directly related to Bitcoin regulation policies for policy events, group together news articles reporting on the same policy events, and assign Tighten, Loosen or Neutral as the policy stance for each policy event. For example, " Chinese Regulators Target Bitcoin in Effort to Limit Capital Leaving the Country" (Wall Street

30 EBSCO provides a title list of the Newspaper Source database at https://www.ebsco.com/products/research-databases/newspaper-source.

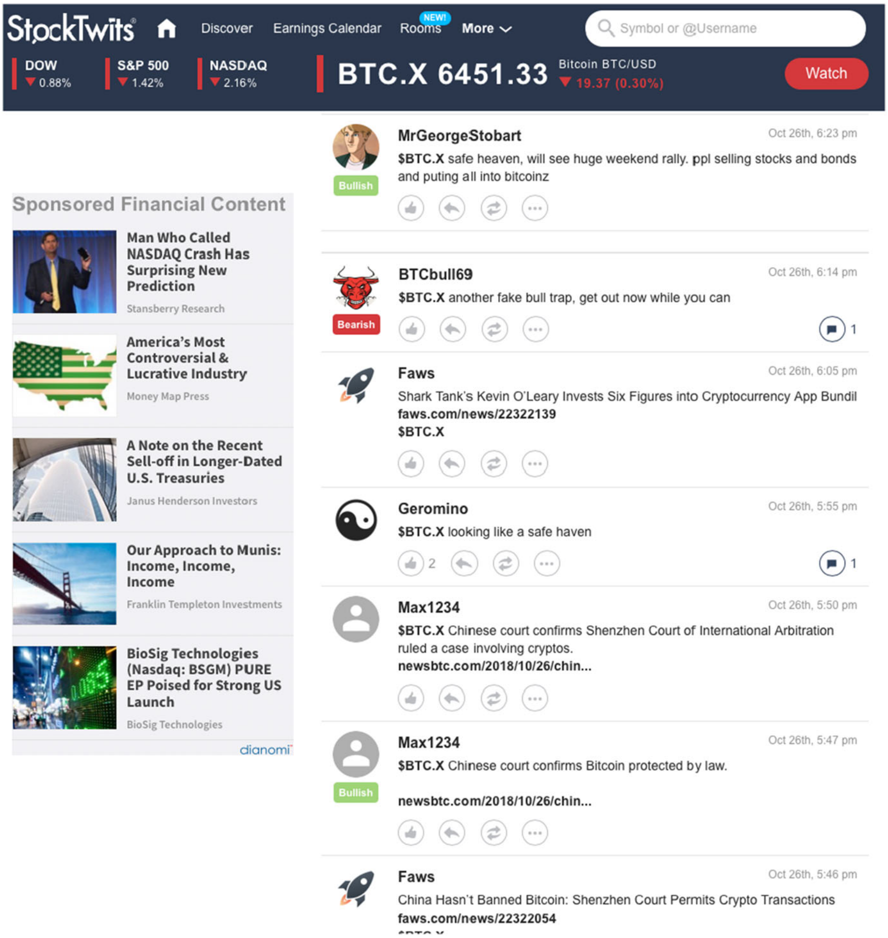


Fig. 8 Illustration of sentiment tagging of tweets on Stocktwits.com

Journal, Jan. 12, 2017) constitutes a tightening policy event, and " South Korea to Expand System for Digital Currency" (Arabia 2000, Oct. 22, 2015) constitutes a loosening policy. When the policy event does not point to a clear tightening or loosening direction, we assign it to be a neutral event. In all, we identify 67 tightening, 14 neutral, and 27 loosening Bitcoin regulation policy events associated with the governments of 24 countries spanning the period of 2013 to 2018.

We then create two measures of Bitcoin policy stance at the daily level using the identified Bitcoin regulation policy events. The first measure is a dummy that indicates whether there is a tightening (loosening) regulation policy event during that day. The second measure is the number of distinct news events regarding tightening (loosening) policies. Theoretically, it is possible to have both loosening and tightening events occurring on the same day. In our sample, the tightening and loosening policy events are exclusive in our sample at the daily level. Finally, the sample average of the tightening day dummy is 0.034, while the sample average of the loosening day dummy is 0.014, which translates to a tightening day on average occurring every 29 days in the sample period, and a loosening day on average is observed every 73 days.

Measures of Bitcoin market sentiment We construct a social media index of sentiment regarding Bitcoin, computed as the ratio of the number of positive tweets to the total number of positive and negative tweets on two platforms, Twitter and Stocktwits. The Stocktwits platform, in particular, allows users to tag "bearish" (negative sentiment) or "bullish" (positive sentiment) when posting tweets. Figure 8 illustrates sentiment tagging of Bitcoin-related tweets on Stocktwits.

To overcome the large amount of noise inherent in high-frequency social media data, we aggregate the tweets data to the bimonthly level. Our negative sentiment measure equals the monthly number of negative messages regarding Bitcoin across Twitter and Stocktwits divided by the number of positive and negative messages. To capture cyclical changes in Bitcoin sentiment, we further pass the Bitcoin sentiment measure through a HP-filter at the bimonthly level with a standard smoothing parameter of 57,600 to remove the trend component.

For historical data on the number of positive and negative tweets on Twitter and Stocktwits, we use data from Decryptz (www.decryptz.com), a cryptocurrency social analysis platform. Decryptz is a product of PsychSignal, a leading social data and sentiment analysis company. Decryptz provides counts of positive, negative and neutral social media messages across Twitter and Stocktwits platforms starting from 2014/09, which we use to compute the monthly measure of Bitcoin sentiment.

Robustness to alternative period definitions In this subsection of Appendix , we report empirical results under the alternative time period length of 10 days or 20 days (baseline is 15 days) (Tables 3, 4, 5, 6).

B Full dynamic system for cryptocurrency equilibrium

1. The capital accumulation K_{t+1} :

$$K_{t+1} = (1 - \delta)\, K_t + I_t. \tag{B.1}$$

2. The stock of cryptocurrency B_{t+1} :

$$B_{t+1} = (1 - \delta_b)\, B_t + \chi_t Q. \tag{B.2}$$

3. The real wage W_t :

$$W_t = (1 - \alpha)\, A_t K_t^{\alpha}, \tag{B.3}$$

4. The real capital return R_t :

$$R_t = \alpha A_t K_t^{\alpha - 1}. \tag{B.4}$$

Table 3 Policy stances and Bitcoin prices (next 10 days)

	(1) Return	(2) Return	(3) Return	(4) Return	(5) Collapse	(6) Collapse	(7) Collapse	(8) Collapse
Tight (dummy)	−0.0457**		−0.0454*		0.183*		0.184*	
	(0.0233)		(0.0235)		(0.0961)		(0.0963)	
Loose (dummy)		0.0239	0.0233			0.0283	0.0348	
		(0.0414)	(0.0433)			(0.0697)	(0.0705)	
Tight (#news)				−0.0387*				0.153*
				(0.0208)				(0.0816)
Loose (#news)				0.0339				0.0495
				(0.0484)				(0.0648)
Constant	0.0255***	0.0236**	0.0252**	0.0249**	0.166***	0.172***	0.165***	0.165***
	(0.0123)	(0.0116)	(0.0099)	(0.0120)	(0.0303)	(0.0318)	(0.0303)	(0.0306)
Observations	1819	1819	1819	1819	1819	1819	1819	1819
R-squared	0.003	0.000	0.004	0.004	0.008	0.000	0.008	0.009

Tight and Loose are news-based measures of Bitcoin regulation policy stances. We use the EBSCO Newspaper Source to search news articles on Bitcoin or cryptocurrencies and government or taxes or regulations. We then manually identify news articles directly related to Bitcoin regulation policies for policy events and assign Tighten, Loosen or Neutral as the policy stance. Tight (#news) equals the number of Tightening policy events in major newspapers on each day, and Tight (dummy) equals one if there are any Tightening events; the Loose measures are similarly defined. Return is the percentage change in the Bitcoin price in the next 10 days. Collapse is a dummy that equals one if, in the next 10 days, there is at least one day with a daily drop larger than 10%

Newey-West standard errors appear in parentheses, $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$

Table 4 Policy stance, sentiment and Bitcoin prices (10-day periods)

	(1) Negative Sentiment	(2) BTC Price (t-1)	(3) BTC Price (t)	(4) BTC Price (t+1)	(5) Collapse (t-1)	(6) Collapse (t)	(7) Collapse (t+1)
Tight (dummy)	0.0337 ***						
	(0.0083)						
Negative Sentiment		0.115	− 1.135**	− 1.345**	0.614	2.590***	0.976
		(0.402)	(0.500)	(0.553)	(0.568)	(0.579)	(0.836)
Constant	− 0.0051	0.0028	3.95E−10	− 0.0041	0.144***	0.144***	0.145***
	(0.0047)	(0.0565)	(0.0526)	(0.0553)	(0.0324)	(0.0312)	(0.0299)
Observations	132	131	132	131	132	132	131
R-square	0.068	0.000	0.042	0.061	0.007	0.117	0.017

Regressions are at the 10-day level. Negative Sentiment equals the period count of negative messages regarding Bitcoin across Twitter and Stocktwits divided by the period count of positive and negative messages, HP-filtered to remove the trend component. In Column (1), we regress negative sentiment on Tight (dummy), a news-based policy stance dummy that equals one if there is any tightening policy event (but no loosening event) in the current period. In Columns (2)–(4), we regress the Bitcoin price on negative sentiment. The dependent variables are the average Bitcoin price in the last period, the current period and the next period, detrended using the same HP-filter. In Columns (5)–(7), we regress a dummy for the Bitcoin price collapse on negative sentiment. The dependent variable (“Collapse dummy”) indicates whether there are any daily drops in the Bitcoin price larger than 10% in the last period, the current period and the next period

Newey-West standard errors appear in parentheses, $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$

Table 5 Policy stances and Bitcoin prices (next 20 days)

	(1) Return	(2) Return	(3) Return	(4) Return	(5) Collapse	(6) Collapse	(7) Collapse	(8) Collapse
Tight (dummy)	−0.0991**		−0.0994***		0.258**		0.261**	
	(0.0407)		(0.0372)		(0.0886)		(0.0890)	
Loose (dummy)		−0.0189	−0.0224			0.152	0.161 *	
		(0.0366)	(0.0412)			(0.0959)	(0.0914)	
Tight (#news)				−0.0745**				0.229***
				(0.0382)				(0.0554)
Loose (#news)				−0.0174				0.177**
				(0.0410)				(0.0773)
Constant	0.0509***	0.0477**	0.0512**	0.0506**	0.281***	0.288***	0.279***	0.279***
	(0.0240)	(0.0233)	(0.0177)	(0.0230)	(0.0442)	(0.0398)	(0.0440)	(0.0442)
Observations	1809	1809	1809	1809	1809	1809	1809	1809
R-squared	0.006	0.000	0.007	0.006	0.011	0.002	0.013	0.015

Tight and Loose are news-based measures of Bitcoin regulation policy stances. We use the EBSCO Newspaper Source to search news articles regarding Bitcoin or cryptocurrencies and government or taxes or regulations. We then manually identify news articles directly related to Bitcoin regulation policies for policy events and assign Tighten, Loosen or Neutral as the policy stance. Tight (#news) equals the number of Tightening policy events in major newspapers on each day, and Tight (dummy) equals one if there are any Tightening events; the Loose measures are similarly defined. Return is the percentage change in the Bitcoin price in the next 20 days. Collapse is a dummy that equals one if, in the next 20 days, there is at least one day with a daily drop larger than 10%

Newey-West standard errors appear in parentheses, $*p < 0.1$, $**p < 0.05$, $***p < 0.01$

Table 6 Policy stance, sentiment and Bitcoin prices (20-day periods)

	(1) Negative Sentiment	(2) BTC price (t-1)	(3) BTC price (t)	(4) BTC price (t+1)	(5) Collapse (t-1)	(6) Collapse (t)	(7) Collapse (t+1)
Tight (dummy)	0.00544						
	(0.0160)						
Negative Sentiment		0.874	−1.311	−1.629	1.390*	4.654***	−0.366
		(1.070)	(1.208)	(1.251)	(0.760)	(0.612)	(1.061)
Constant	−0.0010	0.0036	−1.47E−9	−0.0136	0.258***	0.258***	0.258***
	(0.0050)	(0.0955)	(0.0403)	(0.0794)	(0.0480)	(0.0446)	(0.0436)
Observations	66	65	66	65	66	66	66
R-square	0.003	0.009	0.020	0.035	0.013	0.148	0.001

Regressions are at the 20-day level. Negative Sentiment equals the period count of negative messages regarding Bitcoin across Twitter and Stocktwits divided by the period count of positive and negative messages, HP-filtered to remove the trend component. In Column (1), we regress negative sentiment on Tight (dummy), a news-based policy stance dummy that equals one if there is any tightening policy event (but no loosening event) in the current period. In Columns (2)–(4), we regress the Bitcoin price on negative sentiment. The dependent variables are the average Bitcoin prices in the last period, the current period and the next period, detrended using the same HP-filter. In Columns (5)–(7), we regress a dummy for the Bitcoin price collapse on negative sentiment. The dependent variable ("Collapse dummy") indicates whether there are any daily drops in the Bitcoin price larger than 10% in the last period, the current period and the next period

Newey-West standard errors appear in parentheses, $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$

5. Aggregate output of final goods Y_t :

$$Y_t = A_t K_t^{\alpha}. \tag{B.5}$$

6. Resource constraint:

$$C_t + I_t + X_t + M_t \nu = Y_t. \tag{B.6}$$

7. The number of miners:

$$M_t = \left[\frac{(1-\pi_t) P_t Q}{\nu} \right]^{\frac{1}{2}}. \tag{B.7}$$

8. For the input in cryptocurrency sector X_t :

$$X_t = \frac{1}{a} M_t H_t^*, \tag{B.8}$$

where $H_t^* = \frac{M_t - 1}{M_t^2 a} P_t Q (1-\pi_t)$.

9. The aggregate investment I_t :

$$I_t = [R_t K_t + (1-\delta_b) \chi_t P_t B_t] \int_{\tau_{\min}}^{\tau_t^*} \frac{1}{\tau} d\mathbf{F}(\tau; \sigma), \tag{B.9}$$

10. The (unified) cutoff value τ_t^* for investment:

$$\tau_t^* = \chi_t \tau_{bt}^* + (1-\chi_t) \tau_{ft}^*, \tag{B.10}$$

11. The cutoff value $\left(\tau_{bt}^*, \tau_{ft}^*\right)$:

$$\tau_{bt}^* = \frac{\beta \Lambda_{t+1}}{\Lambda_t} (1-\pi_{t+1}) \left[R_{bt+1} \Phi\left(\tau_{bt+1}^*; \sigma\right) + \tau_{bt+1}^* (1-\delta) \right] + \pi_{t+1} \tau_{ft}^*, \tag{B.11}$$

$$\tau_{ft}^* = \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[R_{ft+1} \Phi\left(\tau_{ft+1}^*; \sigma\right) + \tau_{ft+1}^* (1-\delta) \right], \tag{B.12}$$

where $\Phi(\tau^*; \sigma) \equiv \int \max\left\{ \frac{\tau^*}{\tau}, 1 \right\} d\mathbf{F}(\tau; \sigma)$.

12. Price of cryptocurrency P_t :

$$P_t = (1-\pi_{t+1})(1-\delta_b) \frac{\beta \Lambda_{t+1}}{\Lambda_t} P_{t+1} \Phi\left(\tau_{bt+1}^*; \sigma\right). \tag{B.13}$$

where $\Lambda_t = 1/C_t$ and χ_t is a binary random variable with $\chi_t = 1$ as bubbly state, and $\chi_t = 0$ as bubbleless state.

13. Following the standard literature, the Markov chain is given by

$$\Pr\left(\chi_{t+1}=0|\chi_t=0\right)=1, \tag{B.14}$$

$$\Pr\left(\chi_{t+1}=0|\chi_t=1\right)=\pi_{t+1}. \tag{B.15}$$

For the non-cryptocurrency equilibrium, we have $\chi_t = 0$. In this equilibrium, we have $B_t = X_t = P_t = 0$. Note that, when $\chi_{t-1} = 1$, but $\chi_t = 0$ is realized at the beginning of t, B_t will be not immediately decrease to zero. Instead, $P_t = X_t = 0$ at the transition path, and thus the law of motion of bubble stock is given by $B_{t+1} = (1-\delta_b)\,B_t$, which will be depreciated over time, and eventually converge to zero with $P_s = 0$ for all $s \geq t$.

C Proof of Propositions

Proof of Proposition 1 To solve the optimal problem, we employ guess-and-verify strategy. We consider the equilibrium where the cryptocurrency exists, i.e., $P_t > 0$. We conjecture that the value function $V_t^s\left(K_{jt}, B_{jt}, \tau_{jt}\right)$, $s \in \{b, f\}$, takes the form of

$$V_t^s\left(K_{jt}, B_{jt}, \tau_{jt}\right) = v_{Kt}^s\left(\tau_{jt}\right) K_{jt} + v_{Bt}^s\left(\tau_{jt}\right) B_{jt},\ s \in \{b, f\}, \tag{C.1}$$

The constraint on equity (7) and the investment irreversibility constraint imply that the investment must satisfy

$$0 \leq I_{jt} \leq \frac{R_t K_{jt} - \chi_t P_t\left[B_{jt+1} - (1-\delta_b)\,B_{jt}\right]}{\tau_{jt}}, \tag{C.2}$$

where χ_t is a binary random variable with $\chi_t = 1$ as the bubbly state where cryptocurrencies exist, and $\chi_t = 0$ as the bubbleless state in absence of cryptocurrencies. The Bellman equation (10) can be rewritten as

$$\begin{aligned} & v_{Kt}^b\left(\tau_{jt}\right) K_{jt} + v_{Bt}^b\left(\tau_{jt}\right) B_{jt} \\ & = \max_{\{I_{jt}, K_{jt+1}, B_{jt+1}\}} R_t K_{jt} + \left(\bar{v}_{Kt+1} - \tau_{jt}\right) I_{jt} + \left(\bar{v}_{Bt+1} - P_t\right) B_{jt+1} \\ & \quad + \bar{v}_{Kt+1}\,(1-\delta)\,K_{jt} + P_t\,(1-\delta_b)\,B_{jt}, \end{aligned} \tag{C.3}$$

where

$$\begin{aligned} \bar{v}_{Kt+1}^\kappa &= \frac{\beta\Lambda_{t+1}}{\Lambda_t}\int v_{Kt+1}^\kappa\left(\tau_{jt+1}\right) d\mathbf{F}\left(\tau_{jt+1};\sigma\right),\ \kappa \in \{b, f\}, \\ \bar{v}_{Bt+1}^\kappa &= \frac{\beta\Lambda_{t+1}}{\Lambda_t}\int v_{Bt+1}^s\left(\tau_{jt+1}\right) d\mathbf{F}\left(\tau_{jt+1};\sigma\right),\ \kappa \in \{b, f\}, \\ \bar{v}_{Kt+1} &= \left(1-\pi_{t+1}\right)\bar{v}_{Kt+1}^b + \pi_{t+1}\bar{v}_{Kt+1}^f, \\ \bar{v}_{Bt+1} &= \left(1-\pi_{t+1}\right)\bar{v}_{Bt+1}^b + \pi_{t+1}\bar{v}_{Bt+1}^f. \end{aligned}$$

The second line in (C.3) is obtained by using the law of motion of capital. Since the objective function is linear in investment I_{jt}, the optimal decision follows trigger strategy. That is, there exists a cutoff value $\tau_{bt}^{*} = \bar{v}_{Kt+1}$, such that if $\tau_{jt} < \tau_{bt}^{*}$, the firm would choose the investment $I_{jt} = \frac{R_t K_{jt} + P_t(1-\delta_b)B_{jt}}{\tau_{jt}}$ and $B_{j,t+1} = 0$. If $\tau_{jt} \geq \tau_{bt}^{*}$, the firm opts to not invest, i.e., $I_{jt} = 0$. In this case, the optimal decision of B_{jt+1} implies no arbitrage condition $P_t = \bar{v}_{Bt+1}$.

Putting the optimal investment decision into the Bellman equation yields

$$v_{Kt}^{b}\left(\tau_{jt}\right) = R_t \max\left\{\frac{\tau_{bt}^{*}}{\tau_{jt}}, 1\right\} + \tau_{bt}^{*}\left(1-\delta\right), \tag{C.4}$$

$$v_{Bt}^{b}\left(\tau_{jt}\right) = P_t\left(1-\delta_b\right) \max\left\{\frac{\tau_{bt}^{*}}{\tau_{jt}}, 1\right\}. \tag{C.5}$$

Similarly, Eq. (11) can be rewritten as

$$\begin{aligned}
& v_{Kt}^{f}\left(\tau_{jt}\right) K_{jt} + v_{Bt}^{f}\left(\tau_{jt}\right) B_{jt} \\
&= \max_{\{I_{jt}, K_{jt+1}, B_{jt+1}\}} R_t K_{jt} - \tau_{jt} I_{jt} + \bar{v}_{Kt+1}^{f} K_{jt+1} + \bar{v}_{Bt+1}^{f} B_{jt+1} \\
&= \max_{\{I_{jt}, K_{jt+1}, B_{jt+1}\}} R_t K_{jt} + \left(\bar{v}_{Kt+1}^{f} - \tau_{jt}\right) I_{jt} + \bar{v}_{Bt+1}^{f} B_{jt+1} + \bar{v}_{Kt+1}^{f}\left(1-\delta\right) K_{jt}.
\end{aligned} \tag{C.6}$$

So there exists a cutoff value $\tau_{ft}^{*} = \bar{v}_{Kt+1}^{f}$, such that if $\tau_{jt} < \tau_{ft}^{*}$, the firm would choose the investment $I_{jt} = \frac{R_t K_{jt}}{\tau_{jt}}$, and $I_{jt} = 0$ if otherwise.

Putting the optimal investment decision into the Bellman equation yields

$$v_{Kt}^{f}\left(\tau_{jt}\right) = R_t \max\left\{\frac{\tau_{ft}^{*}}{\tau_{jt}}, 1\right\} + \tau_{ft}^{*}\left(1-\delta\right), \tag{C.7}$$

$$v_{Bt}^{f}\left(\tau_{jt}\right) = \bar{v}_{Bt+1}^{f} = 0. \tag{C.8}$$

According to the definition of $\bar{v}_{Kt}$ and $\bar{v}_{Kt}^{f}$, we can derive the evolution of cutoffs

$$\begin{aligned}
\tau_{ft}^{*} &= \bar{v}_{Kt+1}^{f} \\
&= \frac{\beta\Lambda_{t+1}}{\Lambda_t}\left[R_{t+1}\Phi\left(\tau_{ft+1}^{*};\sigma\right) + \tau_{ft+1}^{*}\left(1-\delta\right)\right],
\end{aligned} \tag{C.9}$$

$$\begin{aligned}
\tau_{bt}^{*} &= \bar{v}_{Kt+1} \\
&= \left(1-\pi_{t+1}\right)\bar{v}_{Kt+1}^{b} + \pi_{t+1}\tau_{ft}^{*} \\
&= \left(1-\pi_{t+1}\right)\frac{\beta\Lambda_{t+1}}{\Lambda_t}\left[R_{t+1}\Phi\left(\tau_{bt+1}^{*};\sigma\right) + \tau_{bt+1}^{*}\left(1-\delta\right)\right] + \pi_{t+1}\tau_{ft}^{*},
\end{aligned} \tag{C.10}$$

where $\Phi\left(\tau^*;\sigma\right) = \int \max\left\{\frac{\tau^*}{\tau}, 1\right\} d\mathbf{F}\left(\tau;\sigma\right)$ is the liquidity premium.

According to the definition of $\bar{v}_{Bt}$, the no arbitrage condition can be further expressed as

$$\begin{aligned} P_t &= \bar{v}_{Bt+1} \\ &= \left(1-\pi_{t+1}\right)\bar{v}^b_{Bt+1} + \pi_{t+1}\bar{v}^f_{Bt+1} \\ &= \left(1-\pi_{t+1}\right)\left(1-\delta_b\right)\frac{\beta\Lambda_{t+1}}{\Lambda_t}P_{t+1}\Phi\left(\tau^*_{bt+1};\sigma\right). \end{aligned} \tag{C.11}$$

Last equation gives the asset pricing formula for the Bitcoin. □

Proof of the existence of bubbleless steady state

Corollary 1 *Under the condition* $\int_{\tau_{\min}}^{\tau_f^*} \frac{1}{\tau} d\mathbf{F}\left(\tau;\sigma\right) < \frac{1}{\alpha}$, *there exists a unique bubbleless equilibrium.*

Proof It is straightforward that the right-hand-side in (30) is strictly increasing in τ_f^*. Meanwhile, it is always held that

$$\lim_{\tau_f^* \to \tau_{\min}} \frac{\int_{\tau_{\min}}^{\tau_f^*} \frac{\tau_f^*}{\tau} d\mathbf{F}\left(\tau;\sigma\right)}{\int \max\left\{\frac{\tau_f^*}{\tau}, 1\right\} d\mathbf{F}\left(\tau;\sigma\right)} = 0 < \frac{\beta\delta}{1-\beta\left(1-\delta\right)}, \tag{C.12}$$

and

$$\lim_{\tau_f^* \to \tau_{\max}} \frac{\int_{\tau_{\min}}^{\tau_f^*} \frac{\tau_f^*}{\tau} d\mathbf{F}\left(\tau;\sigma\right)}{\int_{\tau_{\min}}^{\tau_{\max}} \max\left\{\frac{\tau_f^*}{\tau}, 1\right\} d\mathbf{F}\left(\tau;\sigma\right)} = 1 > \frac{\beta\delta}{1-\beta\left(1-\delta\right)}. \tag{C.13}$$

Therefore, (30) must have unique solution for τ_f^*. Furthermore, R_f can be directly obtained from (29). Once τ_f^* and R_f are solved, the capital stock can be immediately obtained $K_f = \left(\frac{\alpha}{R_f}\right)^{\frac{1}{1-\alpha}}$. The aggregate output is $Y_f = \left(\frac{\alpha}{R_f}\right)^{\frac{\alpha}{1-\alpha}}$. The aggregate consumption is

$$C_f = Y_f - \delta K_f = \left(\frac{R_f}{\alpha} - \delta\right) K_f. \tag{C.14}$$

A positive consumption requires $R_f > \alpha\delta$, which implies $\int_{\tau_{\min}}^{\tau_f^*} \frac{1}{\tau} d\mathbf{F}\left(\tau;\sigma\right) < \frac{1}{\alpha}$.

Proof of Proposition 2 From the analysis in Sect. 4.3, the cutoff in the bubbly equilibrium τ_b^* is uniquely determined by (31) under the condition $\tau_{\max} > \frac{1}{(1-\pi)(1-\delta_b)\beta}$.

Given τ_b^* and τ_f^*, R_b can be solved from (B.11),

$$R_b\left(\tau_b^*, \tau_f^*\right) = \frac{\left[\frac{1}{\beta(1-\pi)} - (1-\delta)\right]\tau_b^* - \frac{\pi}{1-\pi}\tau_f^*}{\Phi\left(\tau_b^*; \sigma\right)}, \tag{C.15}$$

which strictly increases with τ_b^*, and decreases with τ_f^*.

Substituting last equation into aggregate investment (24) yields an implicit function for τ_f^* (note that $PB = 0$)

$$\frac{\beta\delta}{1-\beta(1-\delta)} = \frac{\tau_f^* G\left(\tau_f^*; \sigma\right)}{\Phi\left(\tau_f^*; \sigma\right)}. \tag{C.16}$$

From (B.4), we can immediately obtain the capital stock $K_b = \left(\frac{\alpha A}{R_b}\right)^{\frac{1}{1-\alpha}}$. From (B.9), the market value of cryptocurrency, PB, can be solved as

$$PB = \frac{K_b\Delta}{1-\delta_b}, \tag{C.17}$$

where the term $\Delta\left(\tau_b^*, \tau_f^*\right)$ satisfies

$$\Delta\left(\tau_b^*, \tau_f^*\right) \equiv \frac{\delta}{\int_{\tau_{\min}}^{\tau_b^*} \frac{1}{\tau} d\mathbf{F}(\tau; \sigma)} - R_b\left(\tau_b^*, \tau_f^*\right). \tag{C.18}$$

Note that the term $\Delta\left(\tau_b^*, \tau_f^*\right) K$ captures the gap between the effective real investment and the internal funds. It can easily verify that $\Delta\left(\tau_b^*, \tau_f^*\right)$ strictly decreases with τ_b^*. Since the evolution of bubble implies $B = \frac{Q}{\delta_b}$, from the last equation, the equilibrium P can be determined as

$$P = \frac{\delta_b K_b \Delta\left(\tau_b^*, \tau_f^*\right)}{(1-\delta_b) Q}. \tag{C.19}$$

The existence of bubble requires $P > 0$, or equivalently,

$$\Delta\left(\tau_b^*, \tau_f^*\right) \equiv \frac{\delta}{\int_{\tau_{\min}}^{\tau_b^*} \frac{1}{\tau} d\mathbf{F}(\tau; \sigma)} - R_b\left(\tau_b^*, \tau_f^*\right) > 0. \tag{C.20}$$

We now show that the condition $P = 0$ implies $\tau_b^* = \tau_f^*$. It is easy to verify that

$$\Delta\left(\tau_f^*, \tau_f^*\right) = 0. \tag{C.21}$$

Since $\Delta\left(\tau_b^*, \tau_f^*\right)$ strictly increases with τ_b^*, we must have $\tau_b^* = \tau_f^*$ when $P = 0$. It is also straightforward to show that the condition $\tau_b^* = \tau_f^*$ implies $P = 0$. To see this, recall that if $\tau_b^* = \tau_f^*$ we must have $R_b = R_f$. The aggregate investment equation immediately gives $\Delta = 0$ or $P = 0$.

For the parameter set (π, σ) such that $P = 0$, we must have $\tau_b^* = \tau_f^*$. Define $\tau^* = \tau_b^* = \tau_f^*$. From the definition of τ_f^* and τ_b^*, we have

$$\frac{\beta\delta}{1-\beta(1-\delta)} = \frac{\tau^* G(\tau^*; \sigma)}{\Phi(\tau^*; \sigma)}, \tag{C.22}$$

$$\Phi\left(\tau^*; \sigma\right) = \frac{1}{(1-\pi)(1-\delta_b)\beta}. \tag{C.23}$$

The first equation is from the equilibrium condition for the bubbleless equilibrium, and the second one is from the bubbly equilibrium. As we discussed in the main text, the second equation determines the equilibrium cutoff as a function of (π, σ). Since $\Phi(\tau^*; \sigma)$ increases with σ, we must have $\frac{\partial\tau^*(\pi,\sigma)}{\partial\pi} > 0$ and $\frac{\partial\tau^*(\pi,\sigma)}{\partial\sigma} < 0$.

Combining last two equations yields

$$\frac{\delta}{[1-\beta(1-\delta)](1-\pi)(1-\delta_b)} = \tau^* G\left(\tau^*; \sigma\right), \tag{C.24}$$

where $\tau^* = \tau^*(\pi, \sigma)$. Last equation determines the relationship between π and σ such that $P = 0$. Under Assumption 1, the R.H.S decreases with σ and increases with π. Therefore, given a fixed σ, we can solve $\bar{\pi}$ from Eq. (C.24), which increases with σ , i.e., $\frac{\partial\bar{\pi}}{\partial\sigma} > 0$. ◻

D Bitcoin and the aggregate economy

The Bitcoin market is closely related to the aggregate economy. The quantitative analysis discussed in Sect. 5.2 shows that the Bitcoin market may present diverse cyclicality patterns depending on the economic conditions. We empirically document the cyclical relationship between the Bitcoin market and the aggregate economy. In particular, we focus on the largest two economies in the world: China and the U.S. The left panel in Fig. 9 presents the time series of Bitcoin prices and real GDP in China and shows that the Bitcoin market price positively comoves with the real GDP. The correlation between these two variables in China is 0.67. The cyclical relationship between the Bitcoin market and the U.S. economy presents an opposite pattern to that of the Chinese economy. In particular, the right panel in Fig. 9 shows that the Bitcoin market is negatively correlated with aggregate output. The correlation between the Bitcoin price and the U.S. real GDP is −0.42.

To document the dynamic impact of a rise in the Bitcoin price to real output, we conduct a structural VAR analysis. We employ the identification scheme proposed in Barsky and Sims (2011). We identify a structural shock to the Bitcoin price that best explains the variation in future Bitcoin prices. Figure 10 shows that a structural shock

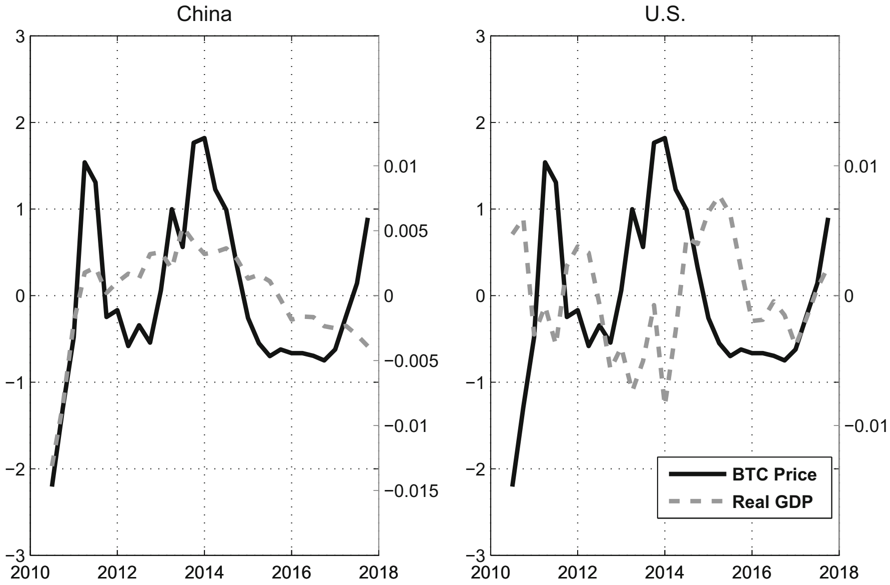


Fig. 9 Bitcoin price and real output. The time series of real GDP and real investment for China are from Chang et al. (2016). The time series for the US are from FRED. All the series are in quarterly frequency from 2010Q3 to 2017Q4. We log-transform each series and detrend them by using the HP filter with a smoothing parameter of 1600

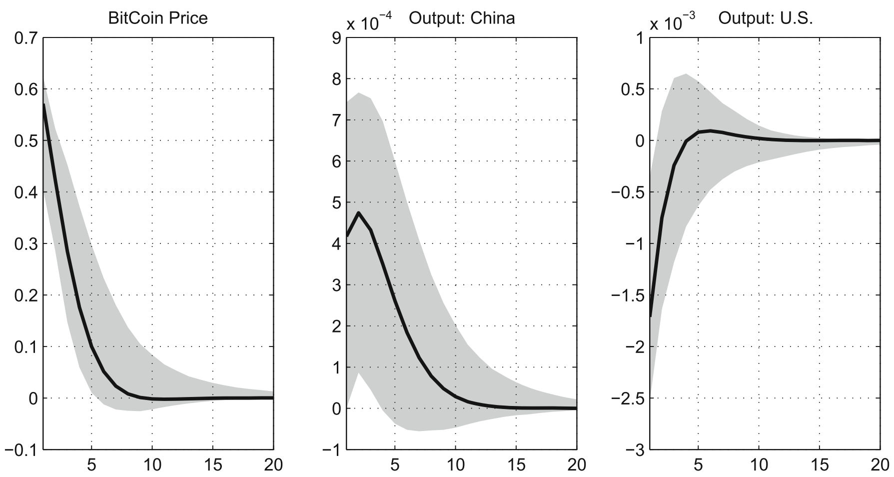


Fig. 10 Impact of Bitcoin price on real output. The VAR system includes three variables in the order of the Bitcoin price, real output in China and real output in the U.S. All three series are HP filtered with a smoothing parameter of 1600. Since the time series are relatively short (only 30 quarters), to avoid the curse of dimensionality, we choose the number of lags to be 1. The solid line is the estimated impulse responses based on the identification scheme in Barsky and Sims (2011). Technically speaking, the shock to Bitcoin prices is identified such that the shock maximizes its share of variance in the Bitcoin price in future k periods. Barsky and Sims (2011) provide more details about the econometric issue. In our exercise, we choose $k = 20$, i.e., 5 years. The main result remains robust to the value of k. The shaded gray area indicates a 68% confidence interval, which is computed from a Monte Carlo simulation with 2000 repetitions

that rises the Bitcoin price leads to an increase in China's output but a decline in US output. This finding confirms the cyclicality pattern found in the previous analysis.

The above empirical findings indicate that the Bitcoin market may have different aggregate consequences on economies with diverse economic conditions, which is consistent with the dynamics predicted by our quantitative model.

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